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AN ACTIVE FILTER PRIMER, MOD 2

BY ARTHUR D. DELAGRANGE

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In the past few years active filters have become very popular. This report explains their popularity as well as what active filters can (and can't) do. It gives the basics of active filter design, both in theory and practice. It can be used as a handbook to build working active filters of the most common types. This report is an update of one issued in 1981.

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FOREWORD

This report gives the basics of active filter design, both theory and practice. It is an update and expansion of a previous report written in 1982. It will be of interest to persons working in the fields of analog filter design or signal processing.

Approved by:

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Sensors and Electronics Division

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INTRODUCTION

This report is an update of a previous edition[1] written in 1982 which, in turn, was preceded by one in 1979. The report has been used as a textbook in an in-house course for seven years. This edition includes some material previously given only in the class. It includes some material not available five years ago. There is a considerable amount not available in textbooks to this author's knowledge, largely drawn from magazine articles. Some text and some illustrations have been changed for better clarity. The graphs in the appendix were done with a digital spectrum analyzer and printer, and are far better for publication than the previous storage oscilloscope photographs. Lastly, some errors have been found and corrected.

In the past 10 years or so, active filters have become quite popular. Probably the biggest single cause is the availability of good monolithic integrated-circuit (IC) operational amplifiers (op-amps). Other factors are improvements in resistors and capacitors (both discrete and monolithic) some advances in the basic theory, new methods for achieving certain functions, availability of computers for analysis and simulation, particularly the better hand-held calculators, and simply the need for more exotic filters as part of more sophisticated systems.

To some extent the term "active filter" has become a magic "buzzword," and misconceptions about them have become widespread. The primary advantage of modern active filters is that they are the most practical solution to a large number of common filter design problems. Most of the theory is not new at all, some of it having been around for more than half a century. The applications are new. In theory, active filters can do no more than passive filters and simple amplifiers can do. The difference is in practicality. Active filters cannot do everything imaginable; there are definite limits. Often on a given project, the filter considerations are left until last and the filter designer is handed an impossible job. There is no such thing as a "best" type of active filter. There are many tradeoffs and the designer must realistically decide what properties he must have and what he can do without.

Active filter design theory could fill a large book, and indeed a number of books have sprung up recently. Some are very good and some are not so good. Table 1 gives the author's preferences. Number one was used as the basis for the original report. Number two is another good, easy to understand book. Numbers three and four are very thorough references on active filters and op-amps, respectively. Some of the material in this report was previously published in numbers five and six. This report is really a companion to number five. Number seven is a classic reference on filter theory, but done entirely for passive

circuits. Number eight is a newer book which can be used to replace number seven. Number nine is one of the better simple "cookbooks." Number ten is a thorough book covering the most recent techniques.

Most of the books attempt to simplify the design to the point where an engineer with no further experience can build a working filter. That will be done in this report, too; however, the less one knows of the theory, the more likely it is that one will run into problems. Therefore, this report will begin with a review of the theory and then proceed to actual designs. The novice reader should not be discouraged if he has trouble with some of the early sections dealing with theory, as later examples will make some points more clear. (On the other hand, the experienced designer will be bored by some of the background material.) Many of the statements will be generalizations or approximations, but all are accurate enough for most engineering design. The filter circuits shown have all been tested and work. The curves shown in Appendix A are all obtained from actual circuits; they are not theoretical or computer-simulation curves.

There are several reasonable assumptions implicit in these or any other filter designs. The filters must be driven from a low impedance; a simple op-amp voltage follower can be added at the input if necessary. A reasonable load is assumed; since all circuits here are arranged to have an op-amp at the output, this means, simply, a load that can be driven by an op-amp. The frequency range must be such that the op-amps perform pretty much like ideal op-amps. Note that an op-amp with a gain-bandwidth of 1 MHz is useless at 1 MHz; the useful range would typically be 10 KHz. And, of course, the allowable op-amp voltage swing must not be exceeded.

This report is limited principally to analog, linear, frequency-domain filters. Although there are many types that fall outside this scope, they comprise a relatively small percentage of actual use. Note that although digital filters are active devices, they are normally considered a separate category and not included in the term "active filters."

This report is hopefully an improvement over the previous editions. It has become more sophisticated, including more options and some new, advance material. The basic approach is unchanged, though, and it should be only slightly more difficult for 2-, 3-, and 4-pole beginners. Whereas the original included only two, three and four-pole versions of the basic filter types this version uses a unified approach where component values are listed in a table and go up to 9-poles. This report gives the basic filters in a more desirable form where capacitance values are all equal and only resistors need be selected; also the need for 3-pole sections is eliminated. Elliptic filters are often mentioned only as an idea; here practical methods for building them are presented. A general method of converting any passive prototype, including bandpass and bandstop, to an active version is given. The original report should be retained, as occasionally the old alternate filter forms might be desired. The op-amp report (source 5) also contains different forms of some filters.

ADVANTAGES AND DISADVANTAGES

Table 2 gives a summary of the characteristics of active filters as compared to passive and digital. Passive filters were the forerunners of active filters, and much of the basic theory carries over. (Crystal and mechanical filters can be reasonably well thrown in with passive filters here.) Digital filters are considered the next step beyond analog active filters, although a direct comparison is not completely fair. For example, a digital filter usually requires an analog prefilter anyway. Many digital filters are derived directly from analog counterparts, and the theory carries over with modification, but many others perform functions one would never attempt with analog circuitry. To confuse the issue farther, there is a class of filters that uses both analog and digital techniques. These will be mentioned at the end of the report.

A word or two of justification for the entries of Table 2. Inductors are the most serious handicap of passive filters. The requirement of a power supply is no longer a significant handicap; but note that active filters require better regulation while digital usually require more power. Passive filters have virtually no high-frequency limit, but become bulky below 10 KHz and virtually unacceptable below 1 KHz. Active filters suit the mid-range from 1 Hz to 1 KHz, but can be used with some care to 10 KHz and in limited applications as high as 1 MHz. Digital filters have no inherent low-frequency limit because the clock can always be slowed down, but are limited on the high end by sample frequency and are generally used below 1 KHz, with limited application to 10 KHz or 100 KHz. Passive filters usually must be impedance-matched on both input and output. Active filters normally have high enough input impedance and low enough output impedance that impedance is not a problem. In digital filters, impedance is not directly relevant; instead one must observe loading rules and compatibility between logic types. Some types of passive filters can have many sections; 23-pole crystal filters are common. Active filter design becomes difficult beyond 10 poles. Digital filters may be expanded nearly indefinitely by adding more hardware (or software) if possible. Similarly, with dynamic range, passive has no inherent limit beyond practical considerations; active is limited by power supply on the high end and semiconductor noise on the low end; digital can be expanded, but note that more bits means more expense and slower operation. Passive tends to be expensive because of inductors, and digital expensive because of the number of parts; however, large-scale integration (LSI) will soon provide exceptions to the latter. Passive filters often show large discrepancies between calculated and actual performance, while active ones tend to be good, especially if tolerance errors are accounted for; and digital is, of course, exact if quantization is included in the analysis. Note that the chief problem with inductors is the resistance of the wire. If the present rapid advance in superconductor technology results in room-temperature superconductors, passive filters will suddenly become much more nearly ideal.

BASIC DEFINITIONS

The reader should be familiar with the basic definitions and facts of this section. Otherwise, difficulties are almost inevitable. Decibels (dB) is defined as 20 times the logarithm (base ten) of a voltage ration. In this report, it is always the ration of the output voltage to the input voltage, equivalent to gain. Attenuation is the opposite of gain, the reciprocal of the voltage ratio or simply the negative number of decibels. Table 3 gives approximate numbers for the most common conversions. By memorizing these few numbers, most other combinations can quickly be estimated mentally. For example, a gain of 100 is 10 squared, so the conversion is 40 dB. An attenuation of 1/5 is an attenuation of 1/10 plus a gain of 2 of -14 dB; a gain of 35 dB is 40 dB - 6 dB + 1 dB, or (100/2)(1.1) = 55; and so forth.

An octave is a factor of two, normally applied to frequency; a decade is a factor of ten. Thus, a function that is directly proportional to frequency (or the reciprocal), such as a simple single-pole filter, has a slope of 6 dB/octave, equivalent to 20 dB/decade.

The radian frequency ω is equal to $2\pi f$ where f is in Hertz (cycles per second). The factor 2π appears often in filter theory; $\omega=1/\tau$ where τ is the time constant of a simple RC; f=1/T where T is the time period of the corresponding sine wave. So again, there is a factor of 2π between τ and T. Unfortunately, the component values work out best in terms of ω or τ, but filter time-domain responses are most meaningful in terms of f or T. For phase, one complete cycle is equivalent to either 360 degrees (°) or 2π radians (rads) and again the factor of 2π occurs between cycles and radians. Phase response is sometimes plotted in terms of group delay, which is the derivative of phase shift with respect to frequency ($d\phi/df$), so a linear phase shift curve corresponds to constant (flat) group delay.

The Laplace transform variable S (also called generalized frequency) is equal to $\sigma + j\omega$, where $j = \sqrt{-1}$. The real part, σ, has to do with damping or exponential decay or energy dissipation and is associated with transient behavior. The imaginary part, $j\omega$, is the same as previously mentioned and has to do with periodic signals and energy storage; in particular to obtain the response of a filter to a sine wave of arbitrary frequency (frequency response), simply take the filter transfer function and substitute $j\omega$ in place of S, implying that any transients have died out and the filter is operating in a steady-state condition.

The usual filter amplitude functions are shown schematically in Figure 1. A low-pass filter retains the low frequencies while rejecting the higher frequencies as completely as possible. A high-pass is the exact opposite, rejecting the lower frequencies. A bandpass retains some band of frequencies while rejecting both highs and lows. A band-reject is the opposite of bandpass, retaining all but a specific band. An all-pass is constant over all frequencies. Here the amplitude is left undisturbed; the properties of interest are phase or time response, and further graphs are necessary.

Figure 2 shows samples of the graphs of filter performance used in this report. The amplitude response versus frequency, also simply called the frequency response, is usually the plot of interest. Either axis may be either linear (lin) or logarithmic (log); always be careful to note which is used in any given graph, as the apparent shape changes considerably when the scale is changed. The best method is log/log, but unfortunately, oscilloscopes are linear on both axes, and oscillators and spectrum analyzers often do not have a log sweep capability. The plots in this report (see Appendix A) are usually log/lin.

The amplitude response (see Figure 2A; low-pass is shown) has three regions. Response in the passband is as flat (constant) as possible. The graph is usually normalized to unity gain (0 dB) although the filter itself may not be. It may have droop toward the edge of the band and/or ripple throughout the band; the latter is usually of constant amplitude, i.e., the peaks and valleys are all a given deviation from the gain at DC. Response in the stopband will eventually approach a straight line on a log/log (or lin/lin) plot. The transition band is simply the region between the two, usually desired to be as steep as possible. Note that we will not quibble that a true high-pass must pass infinite frequency, but tacitly admit that it need pass only up to some range of interest.

The filter cutoff frequency, or simply filter frequency, is the edge of the passband; it may be defined in at least five different ways, which may or may not give the same point, and the user should understand which is used for any given plot. The most common is the 3 dB down point (F_1). However, if the filter is a type that has ripple, the cutoff frequency is better defined as the point the curve leaves the ripple band (F_2), which is usually less than 3 dB. The cutoff frequency may be defined at the intersection of the linear extensions of the curve from the response at low and high frequency, (F_3), as both will eventually become straight lines on a log/log plot. It may be some convenient number that falls out of the mathematical analysis (F_4; not shown). Lastly, it may be defined by phase, even for an amplitude filter; for example, the frequency at which the phase shift is 45° times the number of sections (F_5; not shown). This report in various places uses F_1, F_2, and F_3; F_4 and F_5 are sometimes the same as the one used. Designs are normalized to a cutoff frequency of 1 rad/sec where it makes sense. Amplitude response versus frequency is often the only specification required.

Usually ignored, but of increasing interest especially in feedback, correlation, or direction-determining systems, is the phase response (see Figure 2B). The vertical may be plotted in either degrees or radians, differing by only a scale factor; in phase lag or phase lead, differing by only a minus sign; but in any case, linear. The horizontal may be either linear or log; linear is best, but the graph then may not correspond to the amplitude response. On a log frequency scale, the curve will flatten out to a constant at both high and low ends; however, phase response is usually not of interest in the stopband. Phase lag usually increases with increasing frequency. It will always approach a constant at high frequency. As noted, instead of phase, the plot may be for group delay, which would be the derivative of the curve shown. Actually group delay is more often the quantity of interest, but phasemeters exist and delay-meters do not. Appendix A uses phase. Phase and frequency response together completely define a filter.

The third quantity of interest is the time domain response, also called transient response or step response. Mathematically, the function of primary interest is the impulse response, the time response of the filter to a pulse of infinite amplitude and infinitesimal width. Since the Fourier transform of an impulse is a flat frequency spectrum and a flat phase spectrum, the Fourier transform of the filter impulse response gives the amplitude and phase response; hence the impulse response completely defines the filter. Unfortunately, an ideal impulse cannot be generated in the real world, so instead the step response is usually used. A unit step is the integral of a unit impulse, so the filter response is the integral of the impulse response. Step response also often has direct physical significance.

For a unity gain filter, the output of a low-pass filter will eventually rise to the input value. (Graphs for non-unity are often normalized to unity anyway.) It will always have a finite slope, may exhibit a noticeable delay and/or overshoot/ringing. Filter rise time is often of interest, and may be defined in a number of ways. It may be the point on the curve $(1-1/e)$, $e=2.718\ldots(T_1)$, the 90% point (T_2), the time it crosses unity (T_3), or the time it enters and stays within some arbitrary $(1\pm\varepsilon)$ error band (T_4). Rise times are listed in some references. They are not given here, but they may be obtained approximately from the curves of Appendix A. Step response is generally most meaningfully normalized to $2/\pi$ seconds (sec) for a filter normalized to radian frequency of unity, which is done here. Scale is, by definition, lin/lin.

SPECIFYING A FILTER

A filter problem is typically specified as an amplitude response curve. For instance, a low-pass filter would ideally have rectangular characteristics (Figure 3A): the passband perfectly flat, the transition band infinitely steep, and the reject band having complete rejection everywhere. Such a filter cannot be built, at least not with a finite number of parts. The first step toward a more realistic filter is to add a finite slope (refer to Figure 3A); however, it turns out that this is still unrealizable because the corner is infinitely sharp. If the corner is rounded, the characteristic may be achievable, but there is no indication yet of how to go about it. (Note again that the filter is not completely specified without adding phase response, but we usually don't care, and for the common types there is a direct relationship between the amplitude and phase anyway.) Linear filters have transfer functions specified by polynomials, or more precisely by a ratio of polynomials, such as:

$$H(S) = \frac{S+2}{S^3+3S^2+4S+2}$$

Each term after the first in either the numerator or the denominator creates one bend in the amplitude response curve, which usually corresponds to one reactive component in a circuit. Thus a ratio of finite polynomials implies a finite number of parts. There are techniques for deriving a filter design from the polynomial, but it does not yet imply a realizable design, as the filter may be

unstable or some components with negative values might be required. There are also mathematical tests for determining which polynomials are realizable. Even so, the correspondence between the polynomials and the shape of the curve is not obvious, but requires plotting by hand or by computer. To make the frequency response plot, recall that S must be replaced by $j\omega$; the example polynomial becomes:

$$\frac{(j\omega) + 2}{(j\omega)^3+3(j\omega)^2+4(j\omega)+2}$$

This can be reduced to two terms, one real (having no j's) and the other imaginary (multiplied by j). If amplitude is of interest, the magnitude is found by taking the square root of the sum of the squares of the real and imaginary parts; if phase is required, it is found by taking the arctangent of the ratio of the imaginary part to the real part.

The next step toward realization (for the method used here) is to factor both numerator and denominator into a product of terms. The example above becomes

$$H(S) = \frac{(S+2)}{(S+1)(S+1+j)(S+1-j)}$$

This can always be done (although it may require a computer). The numerator factors are termed zeros and the denominator factors are termed poles. The transfer function becomes zero at the zeros and infinite at the poles. The poles and zeros may be plotted on a two-dimensional plot of real part versus imaginary part, called the pole-zero plot, S-plane or complex plane (Figure 3B). A zero is indicated by a "O" and pole by an "X." (The word complex should be used only to mean "having both real and imaginary parts" and not "complicated"). Both axes are lin. This plot turns out to be useful for several reasons. The poles are the natural frequencies of the filter; they give the modes of oscillation and decay that will occur if the filter is hit with a bunch of energy (such as an impulse) and then left to its own devices. The poles cannot be in the right half-plane, or the natural response would be an indefinitely increasing function instead of eventually dying out. Note then that the transfer function becoming infinite at some (generalized) frequency may or may not imply instability! A practical filter cannot have poles directly on the imaginary ($j\omega$) axis either, as this implies a sinusoidal response that continues forever--an oscillator. In fact filters having poles very close to the imaginary axis may have problems because a small drift (in component values) may cause the pole to move into the right half-plane. In a transfer function, there is no restriction on the placement of the zeros; as long as the response is zero, we don't really care what caused it. The poles and zeros must be placed symmetrically about the real (σ) axis (complex conjugates) so that when the factors are multiplied out, the imaginary parts cancel; otherwise the design would require components having imaginary values (imaginary components?). Since a sinewave at a given frequency represents a point on the imaginary axis, the magnitude of the filter response at that frequency will be the product of the magnitudes of the vectors from that point to each of the zeros divided by the product of the magnitudes of the vectors from that point to each of the poles. The phase angle* is the sum of the vector angles from the zeros

*Angles are measured counterclockwise from the positive real axis.

minus the sum of the vector angles from the poles. As the frequency passes near a pole, the response will obviously become larger because one denominator factor will become small, vice-versa for a zero. If the frequency passes through a zero on the imaginary axis, phase will jump 180°. And so forth. Thus, the response can be mentally estimated to some degree, and the effect of moving one of the poles or zeros can be observed. Frequency may be positive or negative; response will be the same due to the symmetry of the plot and really only half of it need be plotted. The pole-zero plot completely defines the filter except for gain factor, which we will consider unimportant because there are simple techniques for correcting it later. The pole-zero plot also shows how to break down a design into manageable sized sections: Any zero requires a corresponding pole* and any complex pole or zero requires a corresponding complex conjugate. The most complicated section type ever type needed, therefore, would be two zeros and two poles.

An alternate method which is sometimes useful is to separate the polynomial into a sum of terms instead of a product; a partial fraction expansion. The polynomial becomes:

$$H(S) = \frac{1}{S+1} - \frac{1}{2}\frac{(1-j)}{(s+1-j)} - \frac{1}{2}\frac{(1+j)}{(s+1+j)}$$

Again this may be plotted in the complex plane, and is called a pole-residue plot (Figure 3C). The zeros are not shown; instead each pole has associated with it a residue, which is the value of the function at that point if the "infinity" is eliminated by removing the denominator factor. Residues must be complex conjugates so the polynomial will be real when multiplied out. Again filter response may be visualized by vectors, but this now requires adding vectorially while correcting for residues, which is difficult. The pole-residue plot completely describes the filter, although the gain factor may be ignored. Note that Figures 3B and 3C correspond to each other but not to Figure 3A.

Of course, what is actually done in 99% of the design problems is to use a standard filter type that is already well-known and well-documented. These types usually exhibit some special property in frequency, phase or transient response or circuit realization, are named by that property or the man responsible for the polynomial or the circuit design, and often exhibit noticeable patterns in the pole plot. Most texts, including this one, tabulate these filters; but using the tabulations requires another element of theory which is described in the next section.

TRANSFORMATIONS

Because of the large number of variables (characteristic, function, number of sections, frequency, impedance, etc.) involved in filter design, it is impossible to provide a complete catalog of ready-to-use circuits. That would

*for the linear types discussed here

fill a library, so some generalization is necessary. What is done is to give a filter in "normalized" or "prototype" form, typically having a cutoff frequency of 1 rad/sec and an impedance level of 1 ohm. The prototype would never be used directly, but a filter having the desired frequency and impedance may be obtained from the prototype by simple transformations. In some texts, the prototype is always given as a low-pass, and an additional transformation is necessary if high-pass is desired, but both are given in this report. An example of a normalized 1 rad/sec ohm design is shown in Figure 4A.

If all capacitor values (and inductors if any) are reduced by a factor K_F, it is fairly evident that the filter will act the same way as before, only K_F times faster. In other words, the frequency characteristic will have the same shape, but all frequencies will be K_F times higher. This is termed "frequency scaling." Figure 4B shows the same filter scaled up to 10,000 rad/sec (about 1.6 KHz). Already the capacitor values look more reasonable.

If all resistors (and inductors if any) are multiplied by a factor K_I and all capacitors are (divided) by the same factor K_I, it can be shown fairly easily that the voltage transfer ratio will be unchanged. All currents will be K_I times less, but this does not affect any voltage ratio. This is termed "impedance scaling." Figure 4C shows the circuit of Figure 4B scaled up by a factor of 10,000 in impedance. For our purposes, performance is identical for the two. The component values are now entirely reasonable; that is a working filter. These two transformations are all the reader need know to use this report. However, the following additional ones are useful at times, and indeed some were used to generate some of the circuits given.

To change a low-pass function to a high-pass function, S can be replaced by 1/S everywhere in the polynomial. It can be shown that the new prototype circuit can be obtained by replacing each capacitor with an inductor (for this transformation). This is not directly useful because the point of using active filters in the first place was to avoid inductors. Further transformations may or may not be possible to eliminate the inductors. Alternately, in some circuits the resistors and capacitors may be simply interchanged; Figure 4D is the high-pass version of Figure 4A. The component values are inverted because the impedance of a capacitor is inversely proportional to its value; they appear to interchange only because the capacitor values in Figure 4A happen to be reciprocals for this example. Note: Component values are given to four significant figures to prevent possible roundoff errors when making transformations. The usual components for filter design are ±1% tolerance (three significant figures) with ±5% or ±10% (two significant figures) adequate in some applications.

A low-pass may be transformed to a bandpass by replacing S by S+1/S in the polynomial, which replaces each capacitor with a capacitor in parallel with an inductor. This is less useful to us, and there is no equivalent RC transformation. Similarly, a band-reject is generated by replacing S by 1/(S+1/S), equivalent to replacing each capacitor with a capacitor in series with an inductor.

Thevenin's theorem says that any network of sources and resistors may be replaced by a single voltage source in series with a single resistor. An example is shown in Figure 4E; the two circuits act identically at the output for any K_T. Norton's theorem is similar but uses a current source in parallel with a resistor.

Sometimes an inductor may be replaced with a synthetic inductor, a circuit made using op-amps, resistors, and capacitors which looks like an inductor at its terminals. It turns out this works best for cases where one terminal is grounded. If the inductor is "floating" (neither end grounded), a transformation using "super-capacitors," also referred to as D-elements or Frequency-Dependent Negative Resistors (FDNRs)[2] may be possible. Here each inductor is replaced with a resistor, each resistor with a capacitor, and each capacitor with a super-capacitor, an active circuit having an impedance of $1/DS^2$. Each impedance in the filter is simply multiplied by 1/S and the voltage transfer ratio remains unchanged. An example is shown in Figure 4F.

RESONATORS

Usually the easiest way to realize a filter characteristic is to use a combination of standard "building block" circuits. Caution: There is often confusion between filter characteristics (polynomials) and filter circuits (topology). The two are virtually independent. A given basic circuit design might be used to realize several different characteristics by simply changing component values; conversely, a given filter characteristic might be realized using a number of different circuit topologies.

The pole-zero and pole-residue plots indicate that any filter may be built by assembling a number of individual single real axis poles and zeros and pairs of complex-conjugate poles and zeros (referred to simply as pole-pairs and zero-pairs). Caution again: Filters are often specified by the number of sections, but a section may have either one pole or a pole-pair. To make matters worse, a pole-pair is sometimes referred to simply as a pole, particularly in bandpass circuits. Filters are often specified by the number of poles and zeros, but the zeros are sometimes ignored. The number of poles may be also referred to as the "order" of the filter, but this becomes less clear when zeros are added.

Single real-axis poles are realized by simple RC circuits without feedback. Pole-pairs are obtained with circuits termed resonators, second-order active feedback circuits which may also be described in terms of the feedback theory parameters natural frequency and damping ratio. In general, zeros are difficult to generate; few filters use them. Zeros always come with associated poles that must be accounted for in the linear filters given here.

Figure 5A shows the most common circuit for generating a complex pole-pair, attributed to Sallen and Key. The low-pass version is shown; the equivalent high-pass version has the resistors and capacitors interchanged. Either circuit looks somewhat like an oscillator; indeed a resonator may be thought of as an oscillator that can't quite maintain oscillation but exhibits a ringing which dies out.

The universal active filter (Figure 5B), also in slightly different forms known as the state-variable or bi-quad, is so-called because it simultaneously provides low-pass, high-pass, and bandpass outputs. With the addition of a

summing amp, it can also provide a band-reject or all-pass output. It is indeed an oscillator circuit with negative feedback added to damp the oscillation. It is somewhat complicated but is becoming more popular now that a quad of op-amps in a single package is available, sometimes with matched resistors for this specific application.

As indicated earlier a passive inductor-capacitor (IC) circuit may be adapted by synthesizing the inductor. The dotted line in Figure 5C indicates that it actually contains not an inductor but an entire active RC circuit. Figure 5C then provides a high-pass pole-pair. For low-pass the super-capacitor transformation should be used (Figure 5D); again the dotted line indicates not a simple two-terminal component (which is not possible), but an active resistance-capacitor (RC) circuit.

There are methods of making resonators using gyrators, Generalized Impedance Converters (GICs), Negative Immittance Converters (NICs), etc. Often a circuit using these that works in theory will not work properly when built because of the actual limitations of these devices. The theory is also more complicated; none are included here.

GAIN AND IMPEDANCE VARIATIONS

Since a frequency-domain filter by its very purpose removes part of the energy in the signal, the output is often considerably lower in amplitude than the input, so some gain may be desired. All filters given here are arranged to have an op amp at the output, and the op-amp can also provide gain.[3] Figure 6A shows the 2-pole high-pass of Figure 4D modified to have a gain or two (6 dB). The buffer is modified to be a gain-of-two amplifier, and the feedback resistor is changed to an attenuator having the same impedance but a gain of one-half to cancel the op-amp gain (Thevenin equivalent). This particular example is attractive because the resistors in the RC network happen to be all equal, whereas they would not be in the unity gain version. In fact, since the impedance of the negative feedback network is arbitrary and the resistors are equal, a trivial change gives the circuit of Figure 6B where now all five resistors are equal. Figure 6C is the low-pass version having a gain of two. Note that the negative feedback network need not be transformed to a capacitive divider; indeed that would not work as the op-amp requires a small DC input current. The feedback capacitor is now a capacitive divider, which places a capacitive load on the op-amp; most op-amps will tolerate this but some types will not. Again, all capacitors are equal and all resistors are equal. Impedance transformation could be used to make the capacitor values unity (standard value) instead of the resistors.

Another way of obtaining equal capacitance values is to let the amplifier gain be the variable. The circuit of Figure 6D has the same characteristic as Figure 6C, but the gain must be 1.58 for the capacitors to be equal. There is no

simple transformation to obtain this circuit from one of the others; some calculations must be done. The product of the capacitors stays the same. The gain is given by the formula:

$$G = 3 - 2\sqrt{\frac{C_L}{C_H}}$$

where C_L and C_H are the lower and higher capacitance values of the unity gain case (Figure 4A).

In some cases the resonator must be adjusted to tighter tolerances than the available resistors and/or capacitors. That may be done here by adding a potentiometer at the op-amp inverting terminal to adjust gain up or down slightly, and reducing the input resistor somewhat and adding a series potentiometer as a variable resistor to adjust frequency slightly in either direction. If the denominator polynomial for a particular resonator is given as:

$$S^2 + aS + b$$

inject a signal at the frequency

$$\omega_{90°} = \sqrt{b}$$

and adjust the potentiometers alternately for a phase shift of 90° and a gain of:

$$G = \frac{\sqrt{b}}{a}$$

If instead the poles are given as:

$$S = \sigma \pm j\omega$$

convert to polynomial form by transformation:

$$a = 2\sigma$$
$$b = \sigma^2 + \omega^2$$

The resonator may also be configured to have a gain of exactly two.[4]

INDEPENDENT RC SECTION FILTERS

The simplest possible filter is a number of independent single RC sections (Figure 7). This really isn't even an active filter, but we might as well start from scratch. Figure 7A shows a 2-pole low-pass; Figure 7B shows a 2-pole high-pass. Higher orders are not shown because they are obtained simply by adding more sections. (Of course a single pole may also be used.) All sections

normally have the same frequency and, hence, these are sometimes called synchronous filters: A slight improvement in the amplitude response may be obtained by shifting the poles somewhat, but this isn't really worth the effort.

The pole-zero plot for the low-pass is shown in Figure 7C. As stated, all poles are on the real axis, usually at the same point. The poles and zeros for the high-pass are obtained by reflecting the locations through the unit circle. Thus the pole-zero plot for the synchronous high-pass would have the same poles but also an equal number of zeros at the origin (zero on both axes). The low-pass may be said to have its zeros at infinity, but this is inconvenient to draw on the plot and is usually ignored.

The amplitude (frequency) response is shown in Figure 7D. It exhibits just a gradual bend downward. A steeper slope may be obtained by adding more sections, but each section adds another 3 dB of droop at the bandedge. Here is one of the many trade-offs that will be encountered. Amplitude response for the high-pass would be the mirror image if a log/log scale were used.

The phase response (Figure 7E) for the low-pass is an arctangent curve. It is fairly linear near the origin only. At the bandedge it is $\pi/4$ (45°) times the number of sections. Eventually it flattens out to $\pi/2$ (90°) times the number of sections. Phase response for the high-pass would be similar, but would end at a multiple of 2π (=0°) at infinity.

The step response is shown in Figure 7F. In general for a low-pass filter there is a delay before the filter responds, followed by a fairly linear rise, then a settling to the final value (unity for a completely normalized filter). The identical section independent RC exhibits no overshoot or ringing, and is sometimes used where this is critical but the other requirements are not. In fact, it can be shown for a two-pole filter that if the poles are not on the real axis, the step response must have overshoot. The familiar Krohn-Hite* 3200 series[5] has a switch on the back to convert the filter to independent RC. Step response for the high-pass is less useful; it begins at unity and falls to zero, and does exhibit overshoot.

The independent RC section filter is really the crudest possible filter, and is used mostly where filtering requirements are minimal. It is the simplest to design and may be expanded, simply by adding more sections. It is an exact model for some real-life situations, for example, a long transmission system having a number of identical amplifiers. If the amplifiers are AC coupled, each may be thought of as a single-pole high-pass filter. Also, each will have some high frequency limit; usually past some point the response falls off at 6 dB/octave, equivalent to a single-pole low-pass filter.

(The sketches that will accompany each set of circuits are not exact. They are intended to point out particular features of each type, and may not all correspond to the same number of sections. For exact curves the reader should refer to one of the texts or Appendix A. Responses are usually indicated for low-pass only.)

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BUTTERWORTH FILTERS

The most popular type of filter is the Butterworth, or maximally flat. The latter name indicates that this type is as flat as possible at zero frequency. The name may be misleading, for if the requirement is that the filter response be as flat as possible all the way to the cutoff frequency, this is not necessarily the best type.

The prototype circuits for Butterworth high-pass and low-pass are shown in Figure 8. Note that the same basic circuit also serves for Chebyshev and Bessel (and others); only component values will change. Component values are obtained from Table 4; frequency and impedance scaling are then applied. The circuits are not unity-gain as in the previous report; in fact, the gain depends on the characteristic chosen and the number of poles used. This form is used here because it greatly reduces the paperwork. Also, the two capacitors in each stage are equal; in fact, impedance scaling can be done separately for each stage in such a way as to make all the capacitors not only equal but a standard value (i.e., factor of ten). The overall gain is the product of all the stage gains; it can easily be corrected back to unity if desired. Each gain setting network can be scaled to a different convenient impedance value, or each could be a potentiometer (pot), which is often good enough as only the resistance ratio need be stable.

Noise bandwidth is the width of an equivalent ideal rectangular-characteristic filter which would pass equal noise power for white (flat frequency spectrum) noise in. For the ideal filter noise power out would be simply proportional to bandwidth; but for a realizable filter, some power is lost in passband droop and some is added in the imperfect transition and reject bands, so equivalent bandwidth may be either greater or less than unity. (One must also be careful with even-order filters with ripple; if filter gain is set to unity at DC, the peaks will be above unity, increasing apparent noise bandwidth. The reverse is true for odd-order.) Noise bandwidth is meaningless for high-pass.

Characteristics and responses are shown in Figure 9. As indicated, for Butterworth the poles lie equally spaced on a circle (a unit circle for the normalized filter) except that the mirror image poles in the right-half plane are absent. The poles for the high-pass are the same for Butterworth.

The amplitude response is very flat at the low frequency end, but begins to bend downward approaching the edge of the passband and then is down 3 dB at the cutoff frequency. As more poles are added, the response remains 3 dB down at that point, but the cutoff becomes steeper and the passband stays flat closer to the bandedge, approaching the ideal rectangular shape.

The phase response is fairly linear at the lower frequencies, but bends upward near the bandedge. The vertical axis is not labeled because the total amount of phase shift is a function of the number of poles used. The step response shows slight overshoot.

The Butterworth then is a good general-purpose filter, with reasonable amplitude response, fair phase response, and reasonable transient response. It is the most often used type. The Krohn-Hite Model 3200 series[5] has a 4-pole Butterworth characteristic.

A filter having slightly improved transition band steepness is the Legendre filter. In fact the Legendre has the maximum possible steepness without having a rise in the passband, i.e., still be monotonically decreasing. However, it is seldom used and will not be included here.

CHEBYSHEV FILTERS

The Chebyshev (also spelled Tshebycheff or various other ways) filter, also called the equiripple, achieves improved steepness in the transition band at the expense of having ripple in the passband. It is derived in such a way that all peaks and valleys have the same magnitude; hence the second name. Often a filter passband requirement will be that the response simply stay within some fixed limits; the Chebyshev fits this application well. For this reason the cutoff frequency is best specified as the point the curve passes through the ripple level on its way down, hence the question mark on Figure 9B2. In this report it is specified at the ripple point, but some use the 3 dB point. Note also that the ripple may be specified as either peak or peak-to-peak; here it is the latter.

The poles are located on an ellipse (see Figure 9B1); the height/width ratio depends on the amount of ripple specified. The phase response has a rather sharp bend near the bandedge. The transient response has considerable overshoot and ringing. Note that for the normalized filter, one cycle of the ringing takes about 2π seconds.

Since the ripple must be specified, another variable is added, making the Chebyshev more difficult to catalog. Tables 5-8 are provided corresponding to several different values of ripple. Note that the frequency-determining resistors are not equal to unity, and values must be inverted for high-pass. Since the bandedge here is given at the ripple point which is not necessarily 3 dB, the relative bandwidth at the 3 dB point is listed. Note that although there is little difference between the two for the sharper filters usually used, there is considerable difference for the low-order low-ripple cases. Noise bandwidth is correspondingly high for the latter cases; for comparison to other filter types (e.g., Butterworth) one should divide the noise bandwidth value by the 3 dB-bandwidth value.

There is also a characteristic called the inverse Chebyshev which has the ripple in the stopband instead of the passband. This is seldom used and will not be included here. Chebyshev minimizes the least-square error in the passband; a generalization called "least-squares" may be made to incorporate a weighing function to emphasize portions of the passband having more importance than others.

The Chebyshev, then, is used where better transition band steepness is required and passband ripple can be tolerated. Larger ripple means steeper slope, so we have another tradeoff. Note, however, that past the transition band the slope in the reject band is the same as for the Butterworth. This is true in general since the ultimate slope is determined only by the number of poles and zeros. Any two filters having an equal number of poles (and equal number of zeros if used) will ultimately have the same slope. For Chebyshev a wider spread of component values occurs than for the Butterworth. This can cause a problem because high- and low-valued components may be made out of different materials (even within a given type designation) and hence may not track with temperature; however, it is minimized in this design by making the capacitors equal, as resistors are better in this respect.

BESSEL FILTERS

The Bessel, or Thompson, or maximally-linear-phase filter is derived by the technique that is used for the Butterworth except the phase characteristic rather than amplitude is made as linear as possible at zero frequency.

The poles lie approximately on an ellipse again (see Figure 9) but relatively close to the real axis. The amplitude response falls off very gradually. The cutoff frequency is usually taken to be the 3 dB down point for any number of poles (it may instead be specified in terms of phase or time delay); but unlike the Butterworth for a large number of poles, the shape approaches not a rectangular characteristic but a Gaussian "bell" shape. Here the 3 dB point is used (see Table 9).

The phase characteristic is very linear to the 3 dB point and beyond. The step response achieves a fairly sharp rise with little overshoot (less than 1%). (Some texts incorrectly say no overshoot.) In fact, it can be shown that the sharpest possible rise without overshoot is for the Gaussian frequency response. This shape is interesting because it transforms into an impulse response which is also Gaussian. However, this extends to infinity in both directions, so an exact Gaussian is not possible without an infinite time delay.

The Bessel characteristic is used only where phase linearity or transient response is of driving importance and the attenuation characteristic is secondary. The high-pass versions are of lesser interest, as the linear phase characteristic does not transform in a meaningful way and the step response does have significant overshoot. Some Ithaco* filters[6] offer both Butterworth and Bessel characteristics.

There is a class of filters called transitional filters which are a compromise between Butterworth and Bessel. These combine the advantages of both, or the disadvantages of both, depending on how you look at it. These are seldom used and will not be included here. One company[7] markets a type they term Besselworth*

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which appears to be a phase-corrected Butterworth, having a Butterworth amplitude characteristic but good phase linearity. Equiripple (Chebyshev) or least-squares techniques may also be applied to phase linearity. The filter type having the fastest realizable rise with no overshoot is the prolate, an obscure type seldom used. An approximation to the Gaussian filter may be made by expressing the Gaussian as a power series, truncating at a finite number of terms, and is referred to as a Gaussian filter.

ELLIPTIC FILTERS

An extension of the Chebyshev approach is elliptic filters (Figure 10), also called Cauer or double-equiripple, which achieve even better transition-band steepness at the expense of ripple in both passband and stopband. The elliptic is essentially a Chebyshev modified by adding zeros in the stopband. Strictly speaking the number of zeros equals the number of poles, but in practice filters having an arbitrary number of zeros are included in this category. The poles are again all on an ellipse, although not quite the same as for the Chebyshev, and the zeros are all on the imaginary axis. The cutoff frequency may be specified in the same two ways as the Chebyshev, and the amount of passband ripple must again be specified. Additionally, the height of the stopband peaks, called the stopband rejection, must be specified, making this type doubly difficult to catalog. (The valleys are all zero amplitude, which is minus infinity on a log scale.) The response at very high frequencies falls off relatively slowly, or not at all if the number of zeros exactly equals the number of poles. For this reason most designs use an extra pole.

The phase response in the passband is similar to the Chebyshev, but it has discontinuities due to the zeros. A filter requirement may specify that the passband response stay within certain limits and the stopband response reach a certain rejection at a specified frequency, then stay below that limit; the elliptic filter suits such a specification well. This is often encountered with sampled (digital) systems such as spectrum analyzers in which the higher frequencies would be "aliased" or folded back into the passband. The Rockland* Model 753A Elliptic Filter[8] advertises 115 dB/octave, but this is a misleading specification for this type of filter, as the response is not down 115 dB at one octave.

Circuits for elliptic filters are difficult not only to catalog due to the number of variables, but to design, the problem being the creation of the required zeros. Each zero-pair comes with a pole-pair, which must be made to correspond to one of the pole-pairs required. Early active circuits, employing twin-tees which require redundant components, were clumsy. Recent texts point out that elliptic filters may be constructed using universal active filters as the resonator elements, but a conversion must be made between the poles and zeros normally cataloged and the element values required. Also, a large number of op-amps are required.

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A unified approach[9] has been devised which takes advantage of the fact that elliptic filters are easier to design in the passive domain. Furthermore, for passive filters the computer programs have been written,[10] the element values extensively catalogued.[11] The passive low-pass prototype is shown in Figure 11A. The best way to convert the low-pass to an active circuit is to use super-capacitors. Capacitors in the passive prototype are changed to super-capacitors, resistors are changed to capacitors, and inductors are changed to resistors (see Figure 11B). This gets rid of the inductors while giving a configuration where the active elements are grounded. Each element impedance in the passive prototype has been multiplied by 1/S, so the overall voltage transfer ratio is unchanged. Extra resistors are needed to provide DC continuity, which gets lost in the transformation. To convert the passive low-pass to a passive high-pass, inductors become capacitors and vice versa (Figure 11C). The high-pass is converted to an active version simply by synthesizing the inductors, since they are all grounded (Figure 11D); the active circuit within the dotted lines appears at its input as a grounded inductor. Circuit behavior in the saturated state is undefined; the extra resistors here prevent circuit latch-up.

Element values have been assigned so the same table can apply to all four cases. Tables 10-15 give a sampling of designs, taken from Reference 11. Note that among steepness of the transition band, passband ripple, and stopband attenuation, any one may be traded off for any other. The active low-pass has been arranged so all capacitors are equal; this cannot be done for the others.

CONSTANT K-FILTERS

One of the oldest filter types is the constant-K ladder (Figure 12); the simplest form has all sections identical as shown. (See Reference 12 for a more detailed explanation.) These are derived by an approximation technique, not analytical, and are not optimized with respect to any particular variable. The original circuits were passive (Figures 12A and 12B). The dotted lines indicate that more sections can be added, hence the descriptor "n-pole."

The pole-plot is difficult to calculate and is not shown. The amplitude characteristic does have ripples which are not equal; the amount cannot be controlled and depends on the number of sections used. The attenuation slope can be quite steep simply because many poles can be easily added. The cutoff frequency is approximate and is calculated from the component values. It is exact for the 3-pole (Reference 12 mistakenly says 5); higher orders have more attenuation at the cutoff frequency. The phase characteristic is fairly linear most of the way across the passband, but bends sharply upward at the bandedge. The step response exhibits considerable overshoot and ringing; however, it is a fair approximation to a delay line, having a rise time noticeably shorter than the delay time.

The active circuits (Figures 12C and 12D) are similar to the elliptic filters. These designs have been juggled so most of the capacitors and resistors are equal. Since the values on the ends of the ladder are just different by a

factor of two, they too may be obtained using the same components and paralleling. Impedance transformation may then be used to make all capacitances unity. These filters are a general-purpose compromise; they are useful in a large number of applications which simply require a filter with reasonable performance for all parameters but no stringent requirements on any one.

LERNER FILTERS

In the filters covered thus far, there has been an implied trade-off between squareness of amplitude response and linearity of phase. This occurs because they were all "minimum phase," a term which has not been explained yet. Thinking of the pole-zero plot, consider a zero in the left-half plane and its mirror image in the right-half plane. The effect on amplitude response would be the same for either one, as a vector from any point on the imaginary axis will be same length to either one. However, the phase angle is different. It can be shown that the phase angle is always less if the zero(s) are in the left-half plane. A filter having no zeros in the right-half plane is said to be minimum phase. (Remember that poles in the right-half plane are not allowed.) In a minimum-phase filter, the amplitude and phase response are directly related by an equation called the Hilbert transform; improving one characteristic unfortunately always makes the other worse.

For years no one pursued the problem much further, but in 1963 Lerner[13] pointed out that having minimum-phase is not a requirement in most systems, and that if right-half plane zeros are used, good performance may be achieved in both amplitude and phase response. Furthermore, he devised a method for generating a good filter. Lerner's work has been nearly ignored, but has recently been updated;[14] the results will be summarized in this section.

Lerner derived his filters as passive filters; Figure 13A shows his basic design. Figure 13B shows a conversion to an active filter by means of super-capacitors. (The device on the right is a differential summer.) Lerner's method utilizes the pole-residue plot (Figure 13C), not the pole-zero plot. The prototype filter is a bandpass, not the usual low-pass. Basically, the poles are spaced parallel to the imaginary axis, with the sign of the residue alternating. The distance from the axis is half the spacing. At each end of the string are "corrector" or "termination" poles having half the spacing but the same distance from the axis, and half the residue.

The pole-residue plot converts directly to the passive filter (see Figure 13A). Each pole-pair is created by an LC; two sets are driven from opposite sides of a differential transformer to give the alternating signs. (This creates the zeros which are required, although they are not explicitly shown in the pole-residue plot.) The residues are determined by the inductors, which are hence all equal except for the two on the end. The capacitors determine the frequency of the resonators. The filter is terminated on each end

with half the usual matching impedance. The circuit may be transformed to an active circuit in several ways; suffice it to say the one shown (Figure 13B) is the best found by this author.

The Lerner filter has ripple in both amplitude and phase, although the amount is not directly controllable, as decreasing one would increase the other. Theoretically, the amplitude ripple is within 1 dB and the phase ripple is within 5°; in practice errors due to component value tolerances are usually predominant over the theoretical limitations. The amplitude response (Figure 13D) typically falls quite steeply at the edges because ripple is allowed and a fairly large number of poles are normally used. The phase response is quite linear clear past the bandedge, and then does weird things (irrelevant anyway). Step response of the low-pass is much like a delay line due to the good phase linearity, showing a clear delay, quick rise, and considerable ringing. Note the "pre-shoot." The step response of the bandpass is not particularly meaningful, since both high and low frequencies are missing, and is not shown.

An actual circuit for a 12-pole bandpass filter of approximately octave bandwidth is shown in Figure 14. The double op-amp circuits are the super-capacitors; the three op-amps on the right are a standard differential amplifier (summer). The values have been juggled so all capacitors are the same; resonator frequencies are determined by resistances, which are varied in pairs to minimize the range of values required. Again the super-capacitor transformation removes the bias path for the op-amps so dribble resistors must be added. The cutoff frequency points are taken as the corrector (termination) resonator frequencies which may be read directly from the reciprocals of the resistor values in the end resonators; these of course represent peaks in the response so the bandwidth at 1 dB or 3 dB down will be slightly wider.

For the low-pass the poles continue down to and include the real axis with equal spacing (not shown, see Reference 14); hence the lower termination becomes a single real pole. In the circuit (Figure 15) this corresponds to eliminating the super-capacitor from the lowest frequency resonator. Note that the cutoff frequency is not normalized to unity; instead the lowest resonator frequency is normalized to unity, which means the cutoff frequency depends on the number of poles used.

The transformation to high-pass is a little more obscure. The upper corrector pole-pair of the bandpass is replaced with a single real pole of cutoff frequency equivalent to the next pole-pair if it were <u>equally</u> spaced (not shown). In the circuit (Figure 16) this corresponds to eliminating the resistor only in the highest frequency resonator; the frequency-determining resistors in the super-capacitor acquire a square-root over the expected value. Since the Lerner characteristic is derived on a linear frequency scale, the high-pass cutoff on a log scale may appear either very sharp or very gradual depending entirely on where the poles are placed. Note also that linear phase shift cannot be continued indefinitely without an infinite number of stages; phase shift of the high-pass version flattens out to a constant above the frequency of the termination pole. Still, it is the only linear high-pass having anything like linear-phase available.

Lerner filters are to be used when both sharp amplitude cutoff and linear phase response are required. They are practical only when using a fairly large number of poles because they are basically an ideal repetitive design with non-ideal terminations, which makes the filters rather large. However, the design is fairly simple, and the large number of poles insures a sharp cutoff. Stopband attenuation is limited, however, by the accuracy of the differential amplifier, typically about 60 dB. For more detail see Reference 14.

BANDPASS AND BANDSTOP FILTERS

Thus far, except for the Lerner filters, only low-pass and high-pass filters have been discussed. Many texts include the bandpass version of each filter type, but the usual transformations produce circuits requiring inductors. Typically half of the inductors and half of the capacitors are not grounded, so neither the synthetic inductors or the super capacitors used earlier will work. We will need either more complicated circuits or different techniques.

The most common bandpass transformation[15] is to make the substitution

$$S \longrightarrow \frac{S^2 + \omega o^2}{S}$$

in the low-pass filter function. This moves the function (including its mirror image) up to a center frequency of ω_o (on a geometric basis) and compresses it by a factor of two, so the bandwidth is essentially unchanged. It preserves the characteristic properties, e.g., equiripple. In the circuit it adds a parallel inductor to each capacitor and adds a series capacitor to each inductor (see Figures 17A and 17B), with each resonator having a frequency of ω_o. The conversion to bandstop (also called band-reject) (Figure 17C) is complementary; it adds a series inductor to each capacitor and a parallel capacitor to each inductor. Here each new capacitor or inductor has a value equal to the reciprocal of the value of the original inductor or capacitor respectively (same impedance magnitude), and the original inductor or capacitor is then chosen to resonate at the desired center frequency. The example shown is a 1.25 dB, 39 dB stopband elliptic design taken from Reference 9. Center frequency was chosen to be 1 rad/sec, which gives about an octave band.

There are exotic techniques for getting around the problem of floating inductors such as GIC embedding,[16] but these are fairly involved and applicable only to certain forms. It is easier to extend the synthetic inductor design already given to make it floating; Figure 18 gives such a circuit.[17] It is equivalent to a 1-Henry floating inductor; larger/smaller values are obtained by scaling the capacitor in direct proportion. The dotted resistors may or may not be necessary for stability. Substitution of this circuit for each inductor of Figure 17B and appropriately scaled for 10KΩ, 1.0Krad/sec (160 Hz) gives the circuit of Figure 19; the bandstop for the special case of octave-band consists of the same circuit sections simply rearranged.

Figure 20 shows the bandpass circuit performance; the solid line is almost "textbook." The dashed line represents raising the center frequency to 1.6 KHz by reducing the capacitors by a factor of ten; the stability resistors had to be reduced considerably and performance is noticeably degraded. Figure 21 shows the performance of the bandstop version. The dotted line represents the first attempt. The resonator frequencies were mismatched so the high impedance of the shunt parallel resonator was not cancelled by the two series resonators as it should be (see Figure 17C), allowing signal leak-through at the center frequency; slight tweaking gave the solid curve.

This technique, although direct, places difficult demands on the op-amps, as evidenced by the poor performance at even moderate frequencies. At higher frequencies it is desirable to use single op-amp sections realizing minimum-sized pole-zero groups. First we need the transformation giving the locations of the new (bandpass) poles and zeros.[18] It is:

$$S' = \frac{-\sigma \pm j\omega}{2} \pm (\cos\theta \mp j\sin\theta)\sqrt[4]{\left(\frac{\sigma^2-\omega^2-4\omega_o^2}{4}\right)^2 + \left(\frac{\sigma\omega}{2}\right)^2}$$

where

$$\theta = \arctan\left[-\frac{\left(\sigma^2-\omega^2-4\omega_o^2\right)}{4} + \sqrt{\left(\frac{\sigma^2-\omega^2-4\omega_o^2}{4}\right)^2 + \left(\frac{\sigma\omega}{2}\right)^2}\,\right]^{*} \div \frac{\sigma\omega}{2}$$

and the original (prototype)(low-pass) pole (or zero)(pair) was

$$S = -\sigma \pm j\omega$$

The first $\pm$ and the $\mp$ go together (when one is plus, the other is minus). Positive roots are used. Therefore each pole (or zero) in the low-pass becomes a pair shifted by $\pm\omega_o$ in the bandpass.

For this example (elliptic), we need resonator sections providing zeros. The simplest known to this author is also given in Reference 18, and is shown in Figure 22. Alpha (α) and β are voltage dividers, only one of which will be used in a given case. Combining with a well-known bandpass resonator[19] and scaling to an impedance of 10^4 ohms and a radian frequency of 10^4 (1.6 KHz), the overall circuit is shown in Figure 23. Performance is given by the solid line in Figure 24, and is nearly ideal. The dotted line is for moving the center frequency up a factor of ten by reducing the capacitors by a factor of ten, and is only slightly off. Note the expanded scale in the passband.

The basic design may be made to work at even higher frequencies by eliminating the op-amps altogether[20] and using fixed-gain amplifiers instead. Figure 25 shows the resonator section. Figure 26 shows the overall circuit for a center frequency of 160 KHz. Figure 27 shows the performance; it is nearly ideal except the distortion is high enough to give extraneous frequency components, as shown

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*NOTE: if either σ or ω is zero, then $\theta = \frac{\pi}{2}$

by the dotted curve taken with a simple meter. Also, the particular amplifiers used are not gain-stable with temperature, and the characteristic changes noticeably at non-room temperatures.

Band-reject filters can also be made with this technique. The pole/zero transformation is:

$$S' = \frac{1}{\left(\sigma^2+\omega^2\right)}\left[\frac{-\sigma\pm j\omega}{2} \pm (\cos\theta \mp j\sin\theta)\sqrt[4]{\left(\frac{\sigma^2-\omega^2-4\omega_o^2\left(\sigma^2+\omega^2\right)^2}{4}\right)^2 + \left(\frac{\sigma\omega}{2}\right)^2}\right]$$

where

$$\theta = \arctan \frac{2}{\sigma\omega}\left[-\frac{\sigma^2-\omega^2-4\omega_o^2\left(\sigma^2+\omega^2\right)^2}{4} + \sqrt{\left(\frac{\sigma^2-\omega^2-4\omega_o^2\left(\sigma^2+\omega^2\right)^2}{4}\right)^2 + \left(\frac{\sigma\omega}{2}\right)^2}\right]$$

Approximation techniques may also be used. A bandpass may be constructed with a high-pass followed by a low-pass (or vice versa). A band reject may be made by adding the outputs of a high-pass and a low-pass. Unfortunately, the two sections interfere; each destroys the properties of the other, e.g., equiripple. However, in the case of the elliptic, it does so in a special way: ripple may double (at worst). For the bandpass, one simply uses a prototype having half the required passband ripple; for a band-reject, one having 6 dB extra stopband rejection. Figure 28 shows an octave bandpass derived from the 1.25 dB/39 dB prototype. Ripple is 1.75 dB (see Appendix A), considerably better than double that of the prototype. This filter is not optimum in the sense of having unequal ripple and an extra op-amp, but it is easier to compute, construct and test.

These techniques are required for bandwidths on the order of one octave. There are two special cases, which fortunately are easier to handle: If a wide bandpass (much greater than one octave) is required, the signal may be passed through a low-pass and a high-pass in tandem with little interaction of the characteristics. If a narrow bandpass is required (the common problem of picking out one frequency from all others), a special class of circuits called narrowband or tuned circuits exists. Indeed, any resonator circuit theoretically can be made into a tuned circuit, but there are a number of practical problems. Tuned circuits near resonance exhibit large voltage and/or current swings. This is not a serious problem with passive circuits, but an active circuit may go into limiting and the calculated linear characteristic then becomes meaningless.

The pole-zero plot of a bandpass tuned circuit is shown in Figure 29. The poles are much closer to the imaginary axis than the real axis; there is a single zero at the origin. Obviously the amplitude is large when the frequency is in the vicinity of one of the poles. One of the practical problems is that for some circuits a variation in component values or addition of stray capacitance or inductance may move the poles into the right half-plane, meaning the circuit will oscillate. The amplitude response is a single peak at the resonant frequency ω_0, or unity for the normalized case. The response is by definition down 3 dB at the bandedges. However, these circuits are usually not inherently unity gain, hence the question mark in Figure 29D. In fact, the gain is usually high, causing problems. The ratio of the center frequency to the bandwidth $\Delta\omega$ is defined as the "quality factor" or simply "Q."

The phase response is a constant $3\pi/2$ at low-frequency (equivalent to π-/2, but irrelevant anyway as the amplitude is near zero), increasing by $\pi/4$ at the bandedge, through zero at the center frequency, another $\pi/4$ at the other bandedge, and to constant $+\pi/2$ at high frequency (irrelevant again). The response of a narrowband filter to any input whatsoever is essentially a sine wave at its center frequency. The step response is an exponentially decaying sine wave, with each peak smaller than the previous; the vertical axis is not labeled because the amplitude depends on the filter bandwidth.

Many different circuits are given in the texts for narrowband filters; all seem to have drawbacks, especially for high Q. The circuit shown in Figure 29A is one of the most common. The Q is determined by the ratio of the capacitors; it is thus limited by available capacitance values, but is quite stable. The gain is very high for high Q, but an attenuator can easily be added at the input. The center frequency can be adjusted somewhat by adding a potentiometer to one or both of the resistors, with minimal effect on Q.

Several modifications to this circuit are possible, as shown in Figure 29B. The capacitors and resistors in the bridged-tee may be interchanged. This makes the capacitors equal, but the op-amp sees a capacitive load at medium frequencies and a low-impedance resistive load at high frequencies, either of which may make the op-amp unhappy. The circuit may also be put in inverting form, as shown; the high gain is compensated to unity by connecting a large input resistor to the small resistor. Note that putting the circuit in unity-gain form does not relax the requirement on the op-amp; its gain must still be high compared to $2Q^2$. Here the center frequency and Q may both be adjusted by pots, but the adjustments are not independent.

Narrowband filters can also be built using the state variable filter or a passive design with a synthesized inductor. In either case, both center frequency and Q can be adjusted by pots, but adjustability often implies lack of stability.

The band-reject problem is similar to the bandpass, except worse. If a wideband is desired, separate low-pass and high-pass filters may be used and the outputs summed. The narrowband-reject filter is termed a notch (Figure 30); the goal here is to reject one frequency while retaining all others. The pole-zero plot is a pair of zeros on the imaginary axis with a pair of poles close by (on a semicircle, to be precise). All vectors will be nearly equal except in the vicinity of the zeros. The amplitude response is unity except for a dip at the center frequency. Bandwidth and Q are usually defined in terms of the 3 dB-down frequencies (as is done here), but occasionally in terms of the points 3 dB up from the bottom of the notch, which may not be a true zero.

Phase response is near zero at low frequency and near 2π at high frequencies, which is the same. It passes through $\pi/4$ phase shift at the bandedges and jumps from $+\pi/2$ to $3\pi/2$ (same as $-\pi/2$) at the center frequency. The step response is basically a step, as both high and low frequencies are passed, but it does exhibit a decaying ringing because the circuit is, in a sense, a tuned circuit.

Notch circuits have all the problems of the narrowband and have the additional problem that finite-frequency zeros must be realized. The standard circuit is the twin-tee with feedback shown in Figure 30A. The frequency is determined by RC's of equal value (if components are paralleled); components must be matched for good performance. Q is adjustable; note that the resistors affecting Q add up to unity, i.e., they may be a pot. Frequency is essentially nonadjustable, as a triple or quadruple pot with good matching would be required.

Recently it has been pointed out that a similar circuit can be built using a modified Wien bridge.[21] There are several mistakes in the article, but the circuit does work. The circuit (Figure 30B) is quite similar to 30A, but has the advantage that it uses only two equal capacitors and resistors. Contrary to the article, they must be matched. Q is again adjustable. Frequency could conceivably be adjusted with a double pot.

Notch filters may also be built using the state-variable filter or synthesized inductors. The most obvious synthesized inductor circuit turns out to have a maximum input amplitude limit proportional to 1/Q, making it virtually useless in high-Q applications.

ALL-PASS FILTERS

All-pass filters, not surprisingly, are filters that pass all frequencies. The object here is not to alter the amplitude response but to impart a desired phase shift (or time delay), or often to correct for an undesired phase shift that has already occurred elsewhere. There are several types of all-pass filters, as indicated in the pole-zero plots of Figure 31. Usually for each pole there is a mirror-image zero so the vector magnitudes cancel and the gain is unity everywhere. However, this need not be true; the amplitude may not be exactly flat but only an equiripple approximation. The poles and zeros may be spaced in simple pairs along the real axis (Figure 31B) or in conjugate pairs along the imaginary axis (Figure 31C). They may be spaced logarithmically (Figure 31B), usually for case for constant phase; or linearly (Figure 31C), usually the case for constant delay.

The circuit of Figure 31A gives a single real pole and matching zero. The amplitude response (Figure 31D) is perfectly flat. The phase response is an arctangent curve, going from 0° at zero frequency through 90° at the "breakpoint" frequency to 180° at high frequency. The step response is amusing; since the high frequency response is of negative sign, it steps in the "wrong" direction and then recovers exponentially to the "correct" value. The frequency may be adjusted by adding a pot in the RC without affecting the gain. Since the DC gain is set by resistors, a trimming pot must be added at the junction if exactly unity gain is required; this does not affect the frequency. This ease of adjustment is important. There are circuits available which give a pair of poles and matching zeros; the state-variable filter may be used. However, these are more difficult to adjust as there are four variables, requiring four pots. The

Lerner filter may be made into an all-pass by taking the bandpass and applying both the low-pass and high-pass modifications. This gives the equiripple approximation and not true unity gain.

All-pass networks are sometimes used to provide linear phase shift (delay lines). In most cases, though, a linear-phase low-pass can be used. More often they are used to provide constant phase shift, normally 90°. It is difficult to design a single network to maintain a constant phase shift. Often it is acceptable instead to substitute a pair of networks whose overall phase shift is arbitrary but which maintain a fixed 90° phase difference between their outputs, which is easier. (See Reference 22 for an example.) In any case, the design usually consists of cascading a number of sections, each having a single pole or pole-pair.

COMPARISON OF FILTER TYPES

Table 16 gives a direct comparison of filter types (here meaning primarily the characteristic but also, to a lesser extent, the circuit realization) with respect to the various filter parameters. These descriptors are generalizations and perhaps, to some extent, the opinion of the author. Although there are a number of different parameters, in any given application they will fortunately not all be important. This report has gone through the filters by type (column); now we will summarize by comparing them by each parameter (row).

Passband droop is worst in the Independent RC and Bessel. It is variable in the Chebyshev and Elliptic because the amount of ripple is controllable. Note, however, that decreased ripple amplitude implies worse transition band steepness; a 0.01 dB ripple Chebyshev is essentially a Butterworth.

Transition band steepness correlates with passband droop, since the bands are adjacent. Again the Independent RC and Bessel are worst. The Chebyshev has a tradeoff with the amount of ripple. The Elliptic is the best available because of the stopband zeros.

Response in the reject band is normally monotonic (continually decreasing), but the Elliptic is an exception. Note that if the number of zeros equals the number of poles, the reject band response is eventually flat and does not drop off any further. Ultimate attenuation is tied to monotonicity. Thus the Elliptic is poor. The Lerner eventually reaches a "floor" because of the accuracy of the difference amplifier. The constant-K is best simply because a large number of sections can be added. Whether or not the design is all-pole (no zeros) directly affects monotonicity and ultimate attenuation.

Noise bandwidth is often important in signal processing systems where signal-to-noise-ratio (SNR) must be calculated. This parameter is affected by transition band steepness, monotonicity, the ultimate attenuation. Again the

Independent RC and Bessel are poor. The Elliptic can range from poor to good depending on the parameters chosen. The constant-K is generally best, again because a large number of sections may be used and it is monotonic.

Whether or not a filter is minimum, phase is determined by the presence or absence of zeros in the right half-plane. Most are, but the Lerner is an exception. In a minimum-phase filter if either the amplitude or phase is good, the other must be poor, as can be seen by comparing the phase linearity with the transition steepness for the other types. However, it is possible for both to be poor, as the Independent RC shows. The only filter type that does well in both is the Lerner, which is not minimum-phase. It has the best linearity for all frequencies passed with any significant amplitude, but if the phase response is of interest primarily at the lower part of the band, the Bessel is preferable. Transient response correlates closely with phase linearity. Recall that the Bessel exhibits quick rise with minimal overshoot.

Component sensitivity is defined as:

$$S_c^\omega = \frac{\partial\omega}{\partial c} \Big/ \frac{\omega}{c} \approx \frac{\Delta\omega/\omega}{\Delta c/c}$$

That is, the sensitivity of some filter parameter (for example a frequency ω) with respect to some circuit value (say a capacitance C) is the calculated partial derivative of the parameter with respect to the value, normalized by their ratio. For practical purposes (also the way it is measured experimentally), the sensitivity is the percent change that occurs in the parameter when the value is changed one percent. For passive components the ratio should be on the order of unity; in some undesirable circuits it can be tens or hundreds, meaning the design will be very sensitive to component tolerance. For active components the ratio should be near zero, as parameters such as op-amp gain may be expected to vary widely; fortunately, this is usually easy to achieve. The actual calculation from circuit equations can be very tedious; Reference 23 is a sensitivity analysis of a simple bandpass filter which takes 135 pages. The descriptors of sensitivity in Table 16 are a general judgement by the author based on working with the filters. The simpler filters tend to be better with respect to sensitivity.

As indicated in the next line of the table, filters are not usually expandable by adding sections without recalculations. Exceptions are the Independent RC, which doesn't make a very good filter for any number of sections, and the constant-K, where the cutoff frequency varies only slightly. More sections can be added to the Lerner fairly easily, but the cutoff frequency changes directly with each new section added.

Complexity is again a general estimate covering number of parts, number of different values, and circuit design. Cost tends to go right along with complexity. Here the Independent RC finally stands out. The more exotic filters are usually more complicated, and the Elliptic is the worst in this category. High-frequency capability correlates negatively with complexity. Of course any type made to work at high frequencies in passive form.

COMMUTATING FILTERS

Several special types of filters deserve at least passing mention. They have all been around as curiosities for some time, but some have recently become practical due to advanced integrated circuit techniques. They belong to a class which may be termed quasi-linear. To a first approximation the circuits behave as linear filters, but the internal circuit is actually nonlinear, which produces some side effects.

The most common is the commutating filter (Figure 32), usually given as a narrow bandpass. A bank of low-pass filters, shown here as simple RC's, are isolated by input and output switches which connect one at a time into the circuit. The switches "commutate" through the bank at a repetition rate f_o. If the input signal is a sine wave at f_o, each capacitor acquires a DC value equal to the average of that particular segment of the sine wave, and the output is a "stepped" version of the sine wave. If the input frequency is slightly different from the commutating frequency, the "DC" values will change slowly, in fact at the difference frequency. If the difference frequency exceeds the frequency of the low-pass filters, they cannot respond rapidly enough, and the output level drops. In general, the half-bandwidth of the overall filter is equal to the bandwidth of the low-passes; the low-pass characteristic (Figure 32D) is simply moved up and reflected about a center frequency f_o (Figure 32E). For the single-pole switches shown here, the bandwidth is reduced by the number of sections (eight) as each resistor is connected to charge/discharge the capacitor only one-eighth of the time. Alternately, if the input switches connect each RC to ground during the time it is not connected to the input, the (half) bandwidth is unchanged, but the amplitude is reduced by one-eight. The center frequency will be determined by an oscillator and, hence, can be made either extremely accurate or externally variable. The filter shape is determined by the low-passes, and can be closely controlled.

There is obviously quantization noise in the output, but if the sections are matched, it is above the eighth harmonic of the center frequency, and can usually be removed by a simple linear filter. Switching noise is similar. Input and output buffers are often required. The circuit will also pass the second and fourth harmonics. It is not useful above half the switching frequency due to sampling error, known as aliasing or frequency folding.

Switches are normally field-effect transistor (FET) gates, as contained in a mutiplexer IC. A simpler version requiring only one set of switches and one resistor is possible, but the general version shown here allows variations such as using multi-pole filters to give a squarer characteristic, or commutating the input and output switches at a different rate to simultaneously modulate the signal to a different frequency band.[24] The components may be rearranged to give a notch, or the output simply subtracted from the input.

SWITCHED-CAPACITOR FILTERS

Consider the circuit of Figure 33A. When the capacitor is connected to the input, it acquires a charge $V_{in}C_{sw}$. When the switch is changed, the charge is dumped into the output. Average current is charge flow per second, which is then $I_{out} = V_{in}C_{sw}f_{sw}$. On the average, then, the circuit is equivalent to a resistance $V_{in}/I_{out} = 1/C_{sw}F_{sw}$ (Figure 33B). The circuit must operate into a ground, but an op-amp can provide a virtual ground. It must be averaged, but an integrator can do that. The overall circuit, which works quite nicely, is shown in Figure 33C. We need some resistance to limit peak currents, but that is inherent in the FET switches that are normally used anyway. Even though the absolute value of integrated circuit capacitors may vary, the time constant of the integrator depends only on their ratio, which can be closely controlled.

We can take two of these integrators and an ordinary invertor and make a universal active filter. The frequency of the overall resonator will be controlled by the switching frequency; in fact, it is directly proportional. Thus we have a resonator where the frequency is controlled externally and hence may be made either extremely stable or variable. Furthermore, the clock may control several stages, giving a multipole filter with continuously variable frequency, which is difficult (impossible is a better word) to do with linear techniques.

There will be noise present, principally at the clock frequency. This is typically a hundred times the resonant frequency and can be eliminated by a simple linear filter, at least for the low-pass case. It can be a problem for the high-pass case. Sometimes cancellation techniques can be used, since the offending frequency is known and is readily available.

TRANSVERSAL FILTERS

Another class of filters can be built, different in the sense that they are constructed basically from time response properties rather than frequency. The most common is built around a delay line (Figure 34). The different stages are summed with different weights and may have either sign. A pulse input will produce an output seven periods long (in this simple example) whose amplitude in each one is the weighing. Mathematically, the frequency response of the circuit is the Fourier transform of this pulse response. Circuit-wise it should be apparent the circuit will respond to some frequencies more than others. For example, let the weights be of equal magnitude but alternating sign. If the input frequency is half the shift frequency, then the "contents" of the stages will have alternating sign and all stage outputs will reinforce, first in one direction and then the other. For an input frequency much lower than the shift frequency, the contents will be nearly the same and, with alternating sign, will

average to nearly zero. For an input frequency much higher than the shift frequency the signs of the contents would be virtually random, again averaging to nearly zero. This would obviously be a bandpass filter of sorts.

This type of filter has become feasible with the advent of charge-coupled devices (CCDs) the first really practical analog delay line. Transversal filters are especially attractive for two special cases. The first is matched filtering where the transmitter response is the reverse in time of the receiver, achieved simply be reversing the tap weights. The other is linear phase: if the weights are arranged symmetrically about the center tap, then that tap represents the delay and each pair around it represent relative phase shifts that are equal in magnitude but opposite in sign. Hence the phase errors relative to the pure delay cancel.

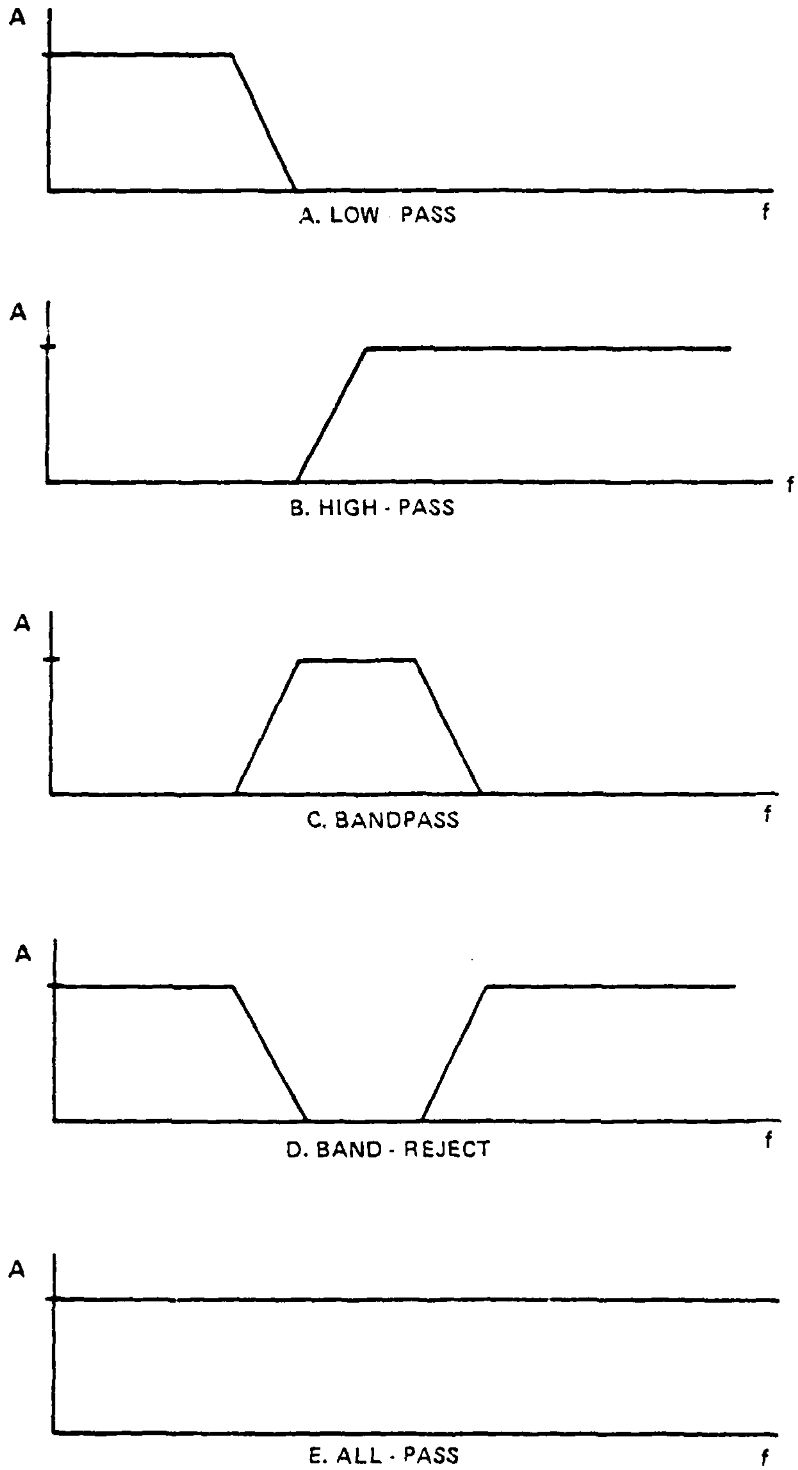
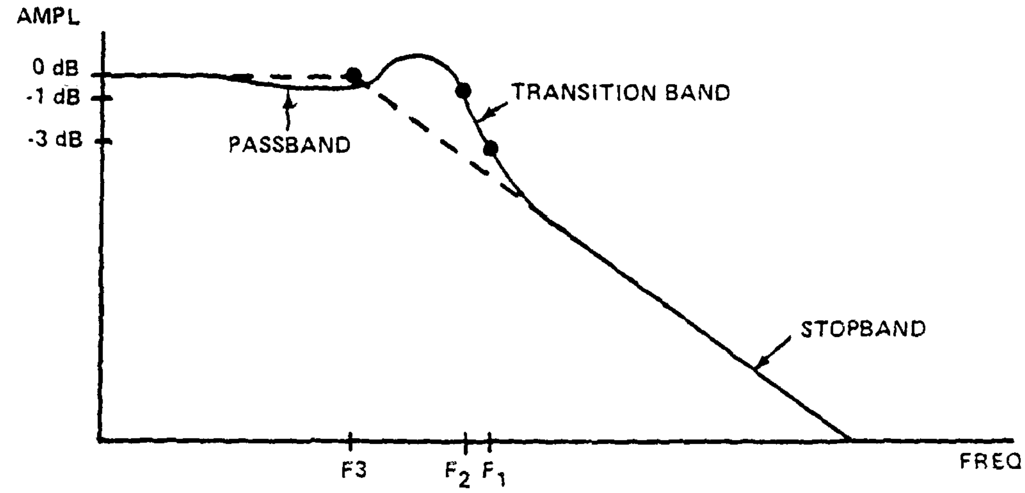
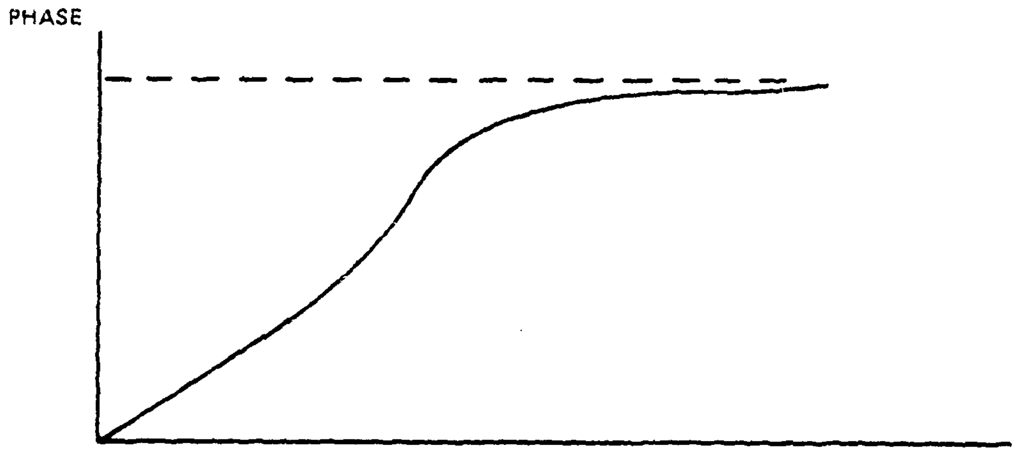


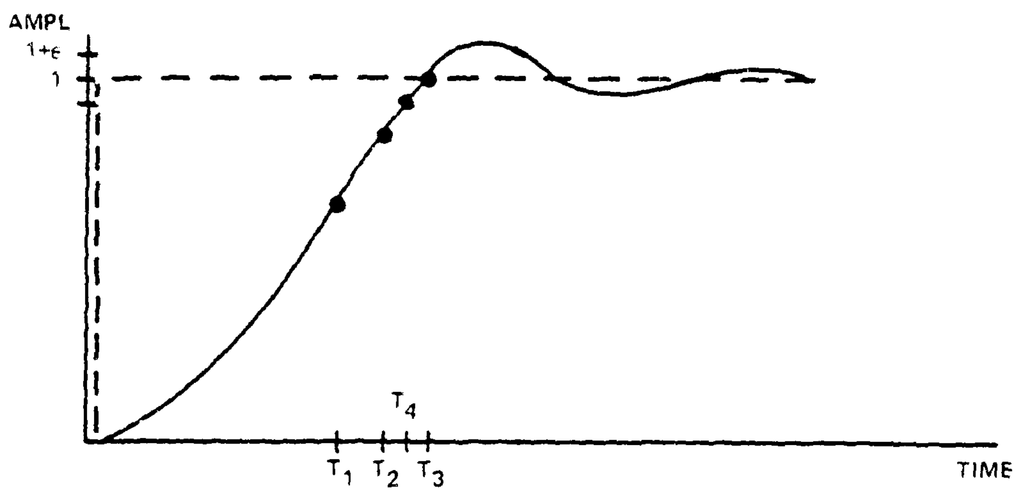
FIGURE 1. COMMON FILTER AMPLITUDE FUNCTIONS



A. AMPLITUDE RESPONSE



B. PHASE RESPONSE



C. TIME RESPONSE

FIGURE 2. GENERALIZED FILTER RESPONSE

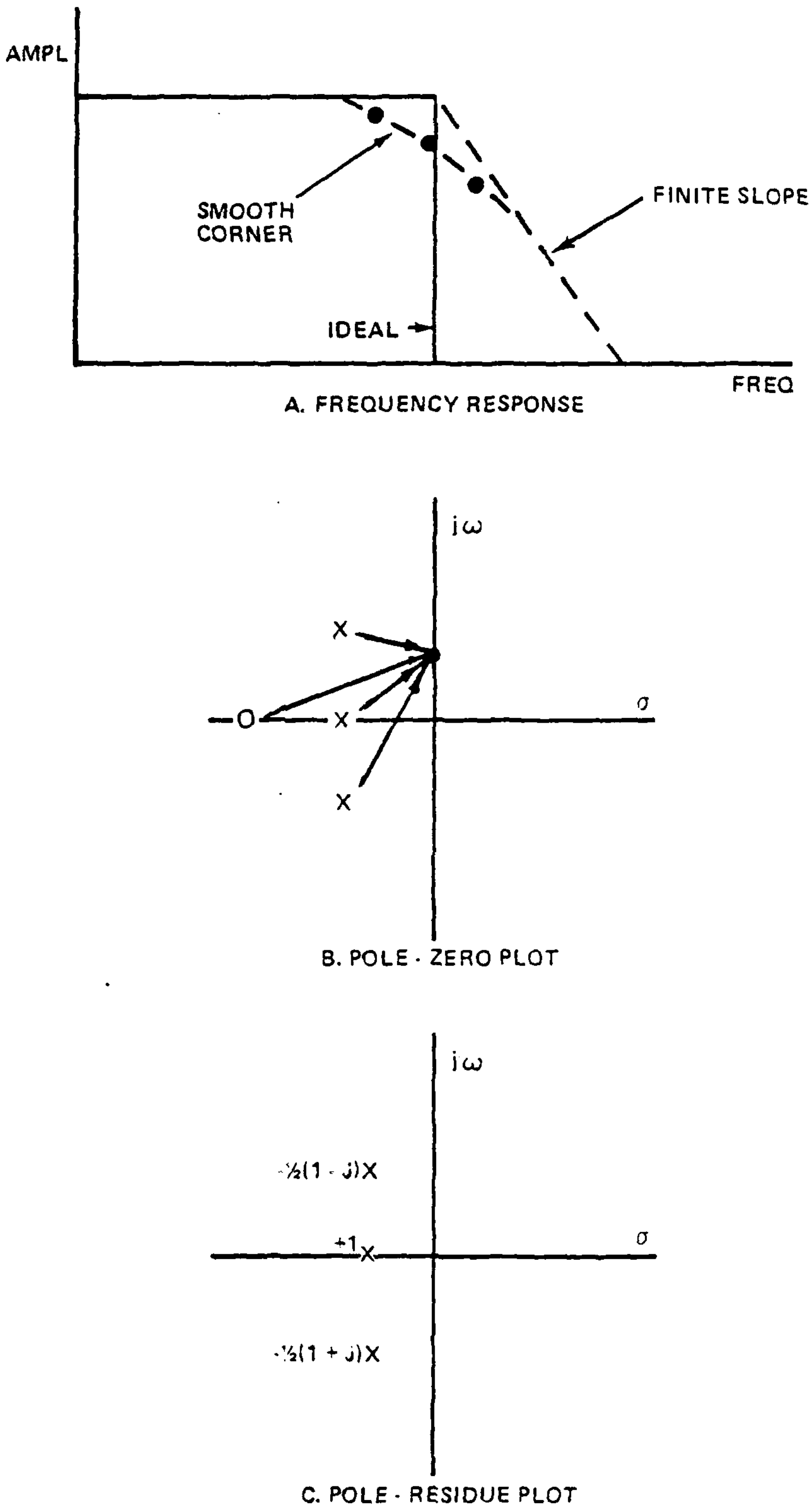
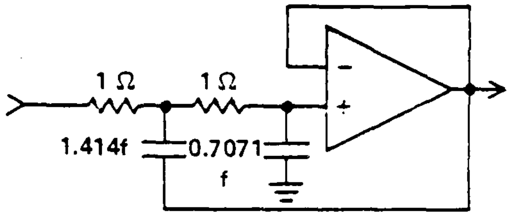
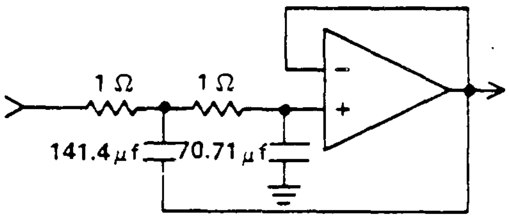


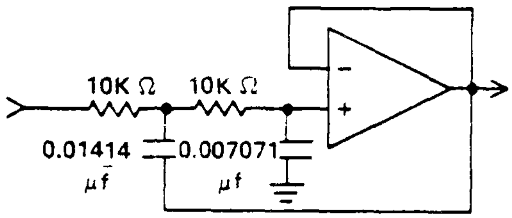
FIGURE 3. GRAPHICAL METHODS OF FILTER SPECIFICATION



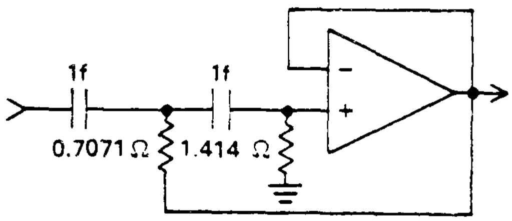
A. NORMALIZED LOW-PASS FILTER



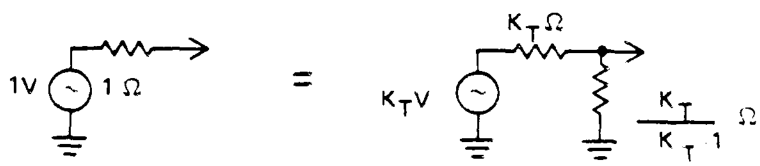
B. FREQUENCY SCALING



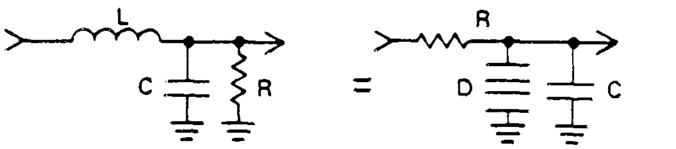
C. IMPEDANCE SCALING



D. HIGH - PASS TRANSFORMATION



E. THEVENIN EQUIVALENT



F. SUPER - CAPACITOR TRANSFORMATION

FIGURE 4. CIRCUIT TRANSFORMATIONS

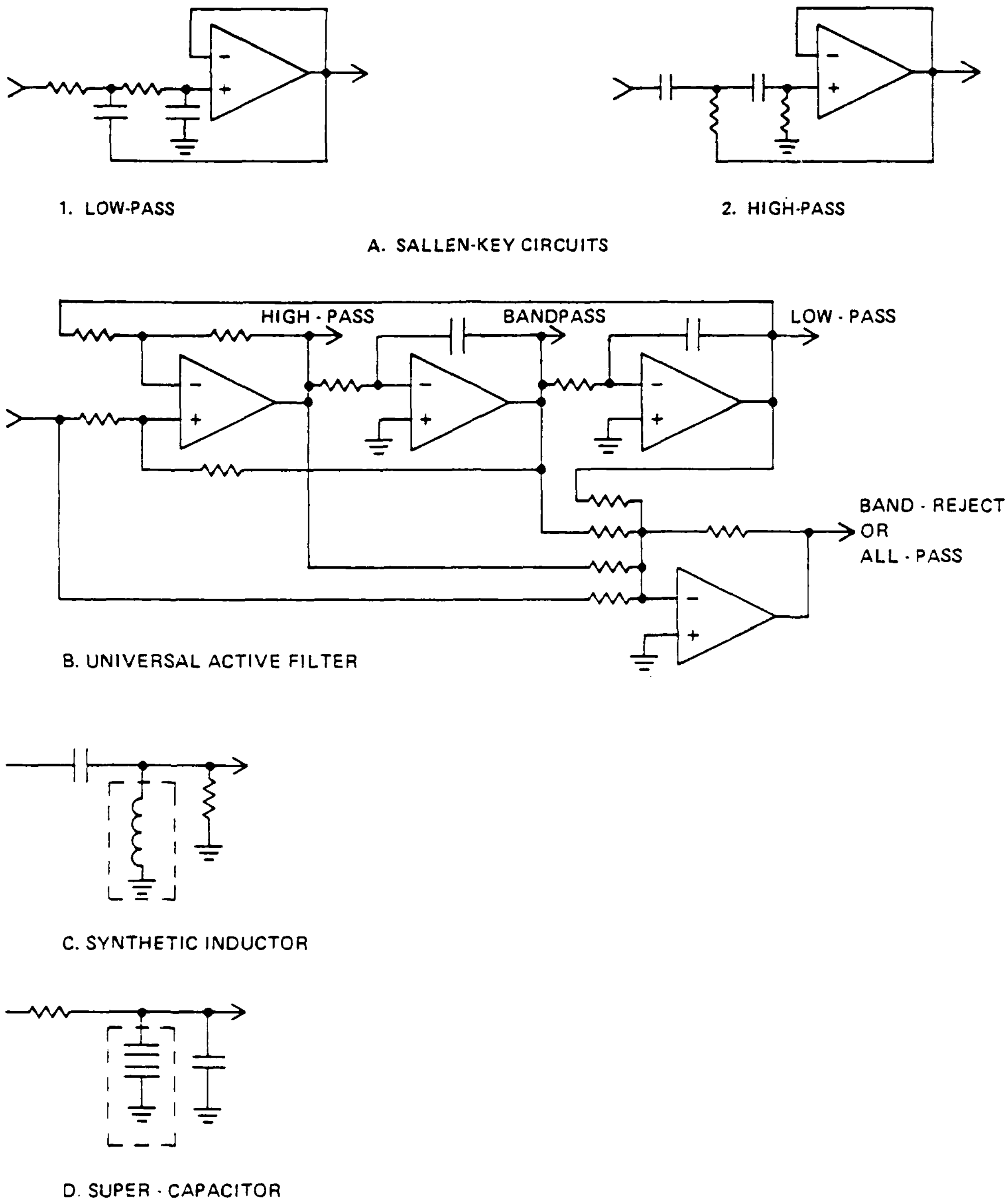
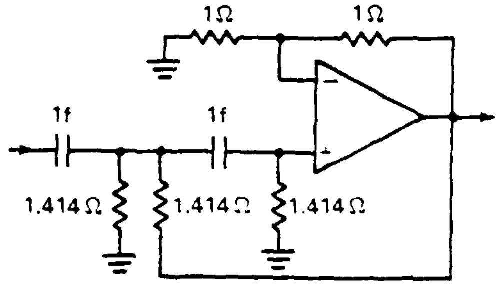
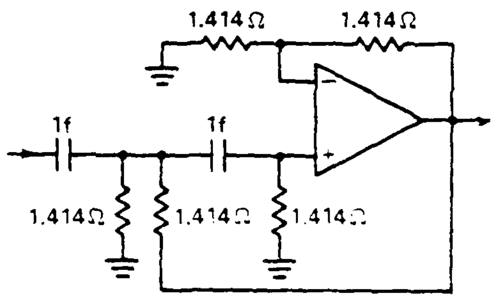


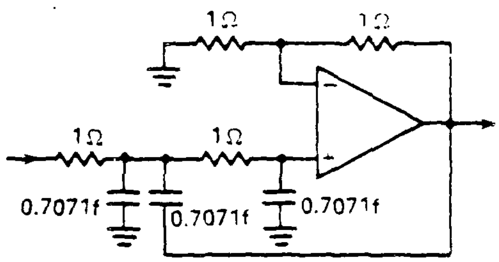
FIGURE 5. RESONATOR CIRCUITS



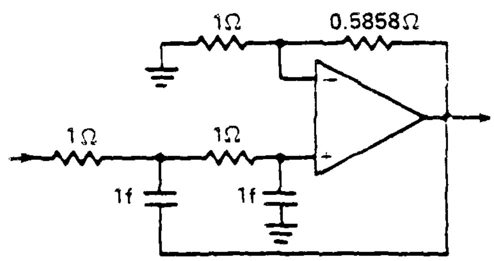
A. HIGH-PASS, GAIN OF 2



B. HIGH-PASS, GAIN OF 2, RESISTORS EQUAL



C. LOW-PASS, GAIN OF 2



D. LOW-PASS, RESISTORS AND CAPACITORS EQUAL

FIGURE 6. GAIN AND IMPEDANCE VARIATIONS

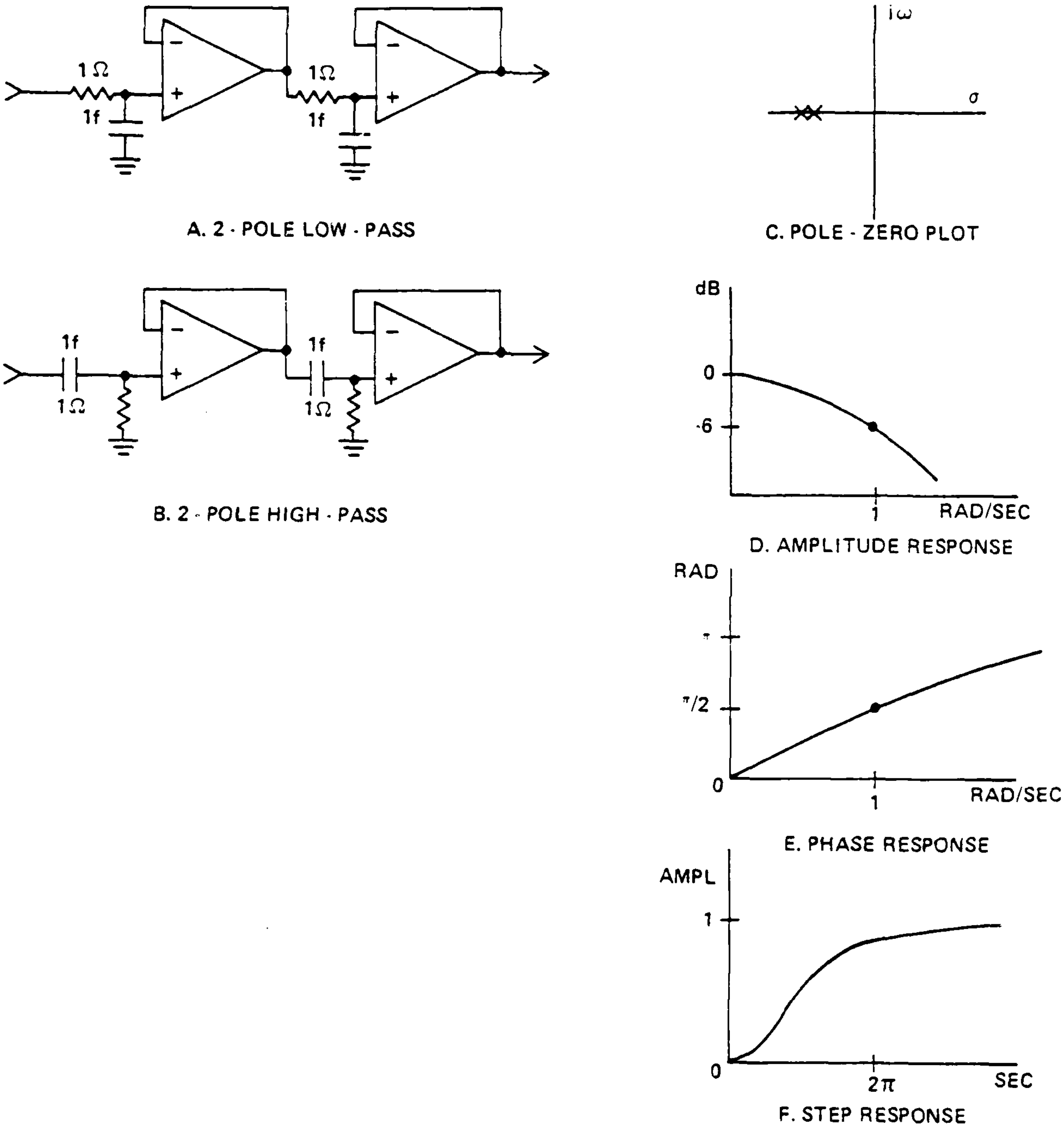


FIGURE 7. INDEPENDENT RC SECTION FILTERS

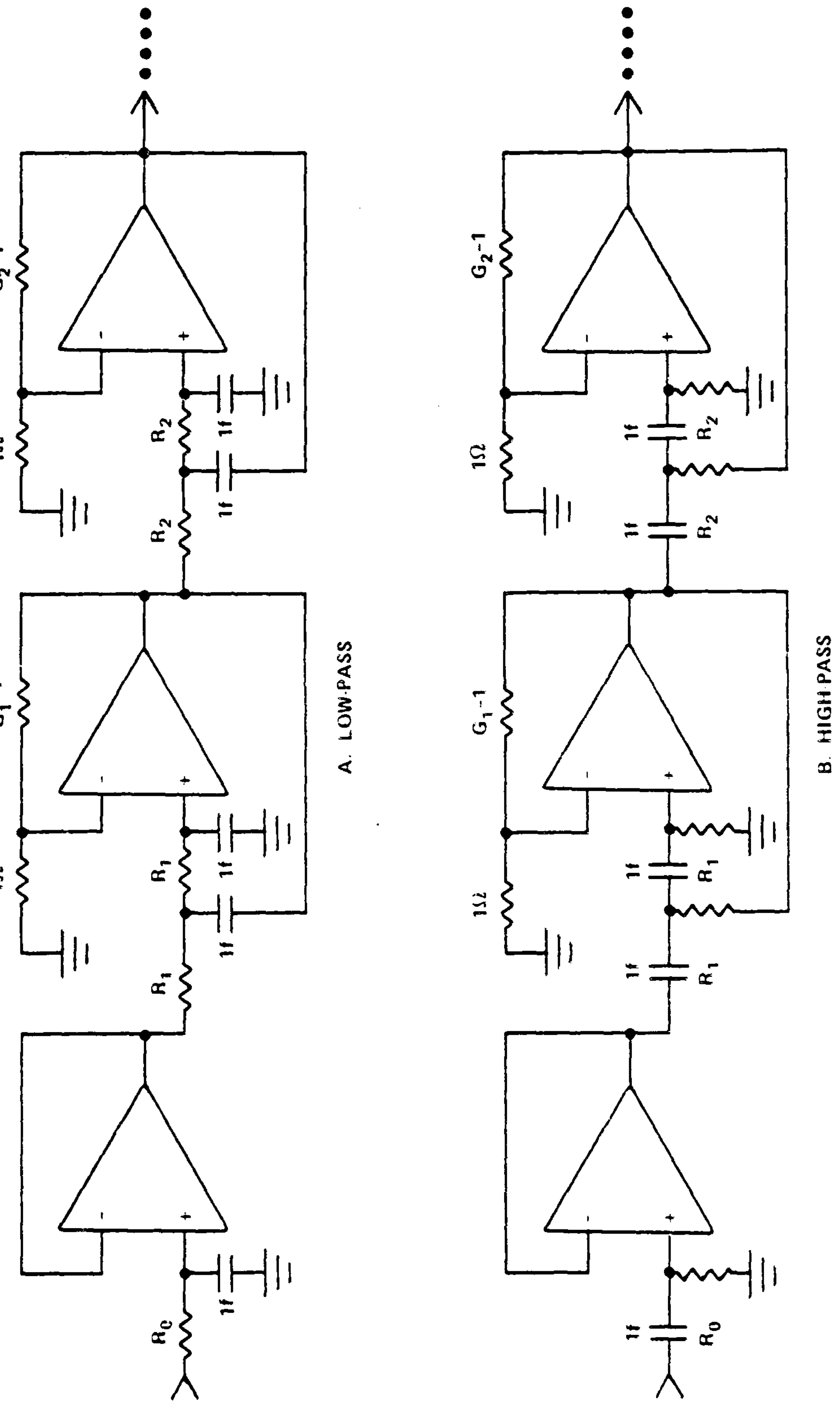


FIGURE 8. PROTOTYPE CIRCUITS (BUTTERWORTH, CHEBYSHEV, BESSEL)

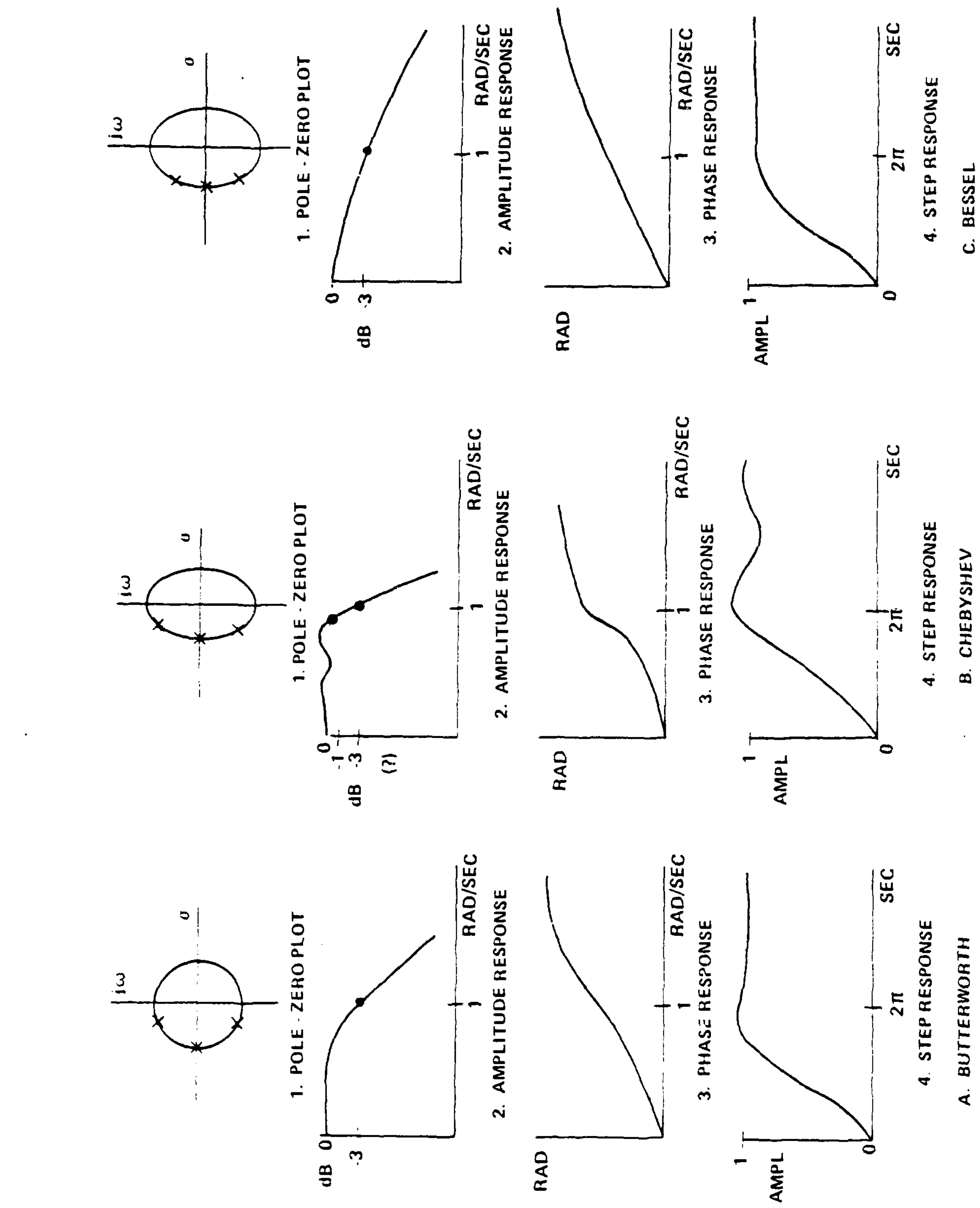
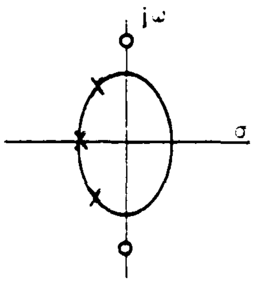
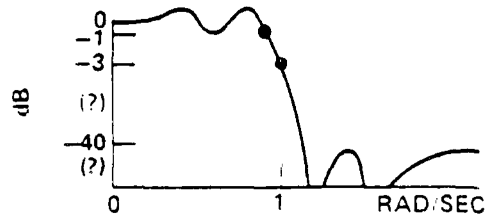


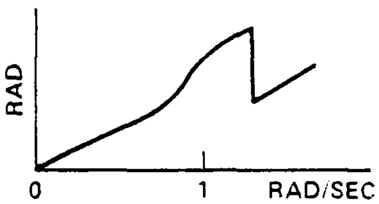
FIGURE 9. FILTER RESPONSES (BUTTERWORTH, CHEBYSHEV, BESSEL)



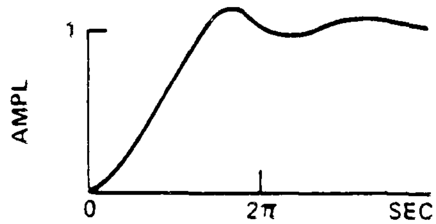
C. POLE-ZERO PLOT



D. AMPLITUDE RESPONSE

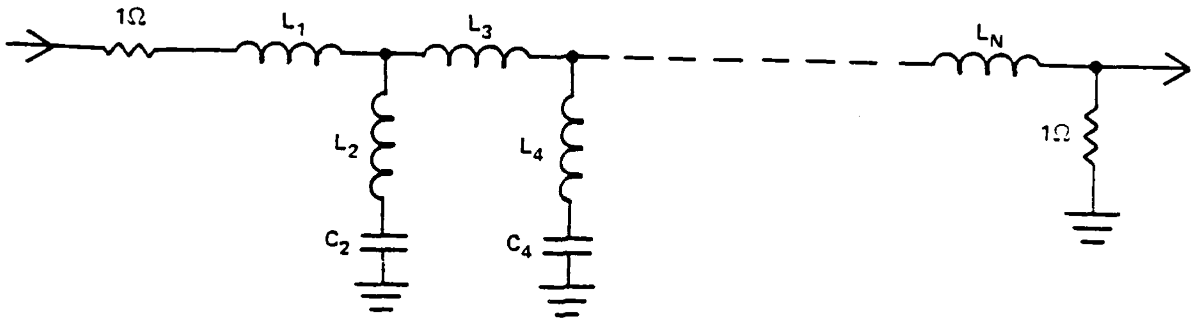


E. PHASE RESPONSE

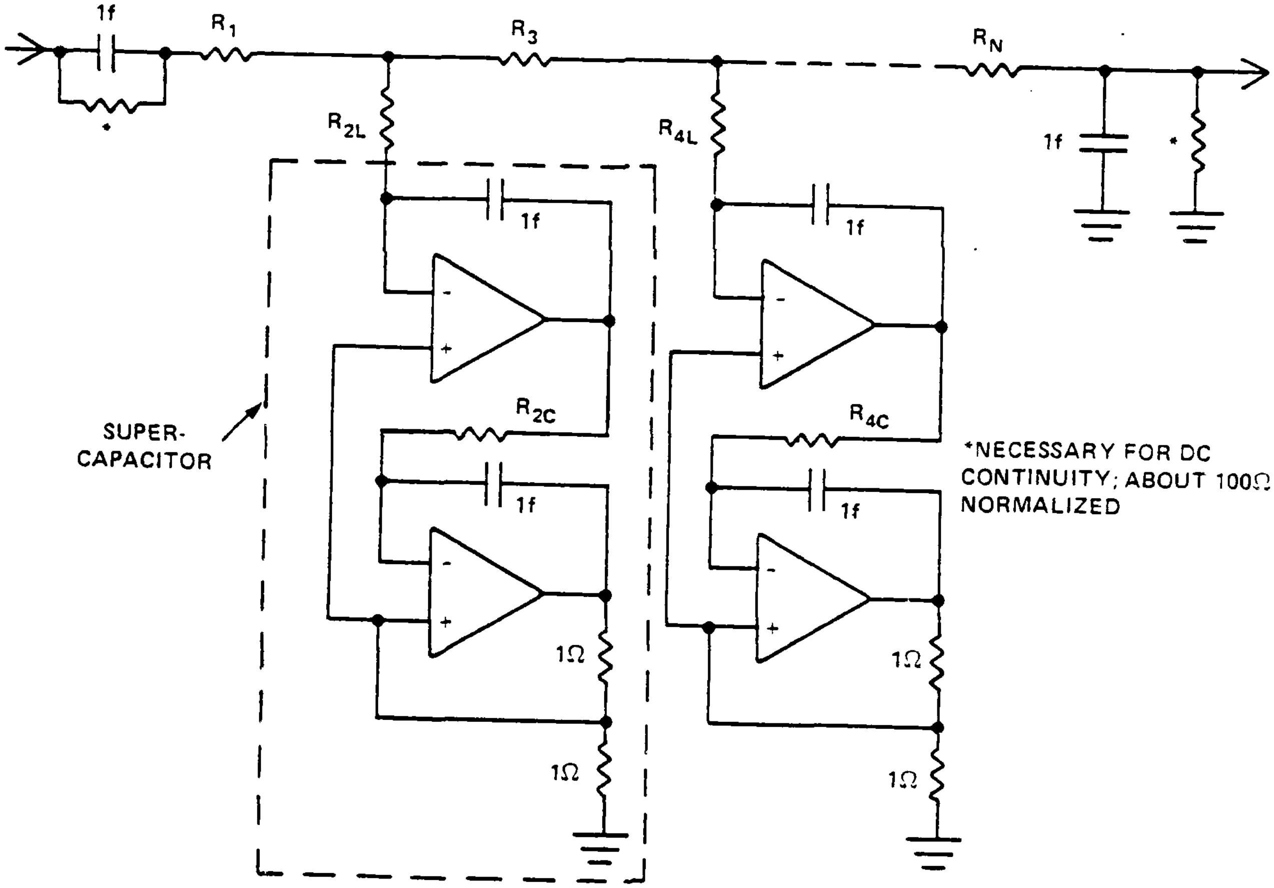


F. STEP RESPONSE

FIGURE 10. ELLIPTIC FILTER RESPONSES

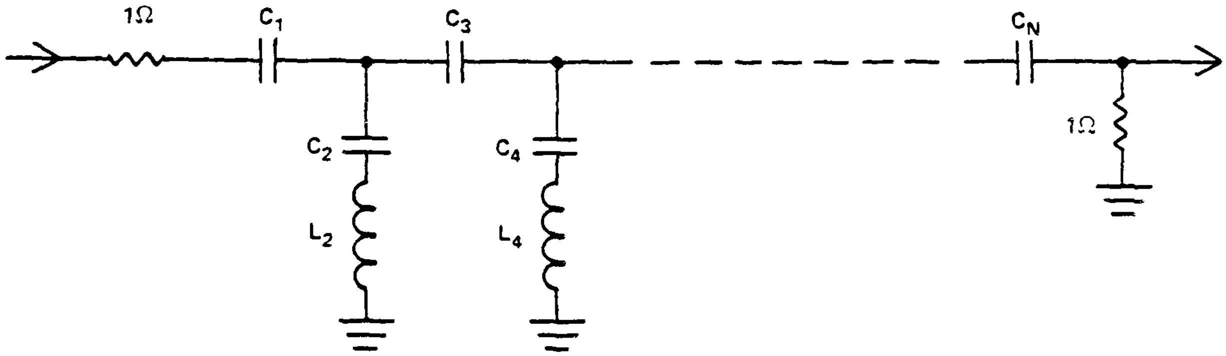


A. PASSIVE LOW-PASS

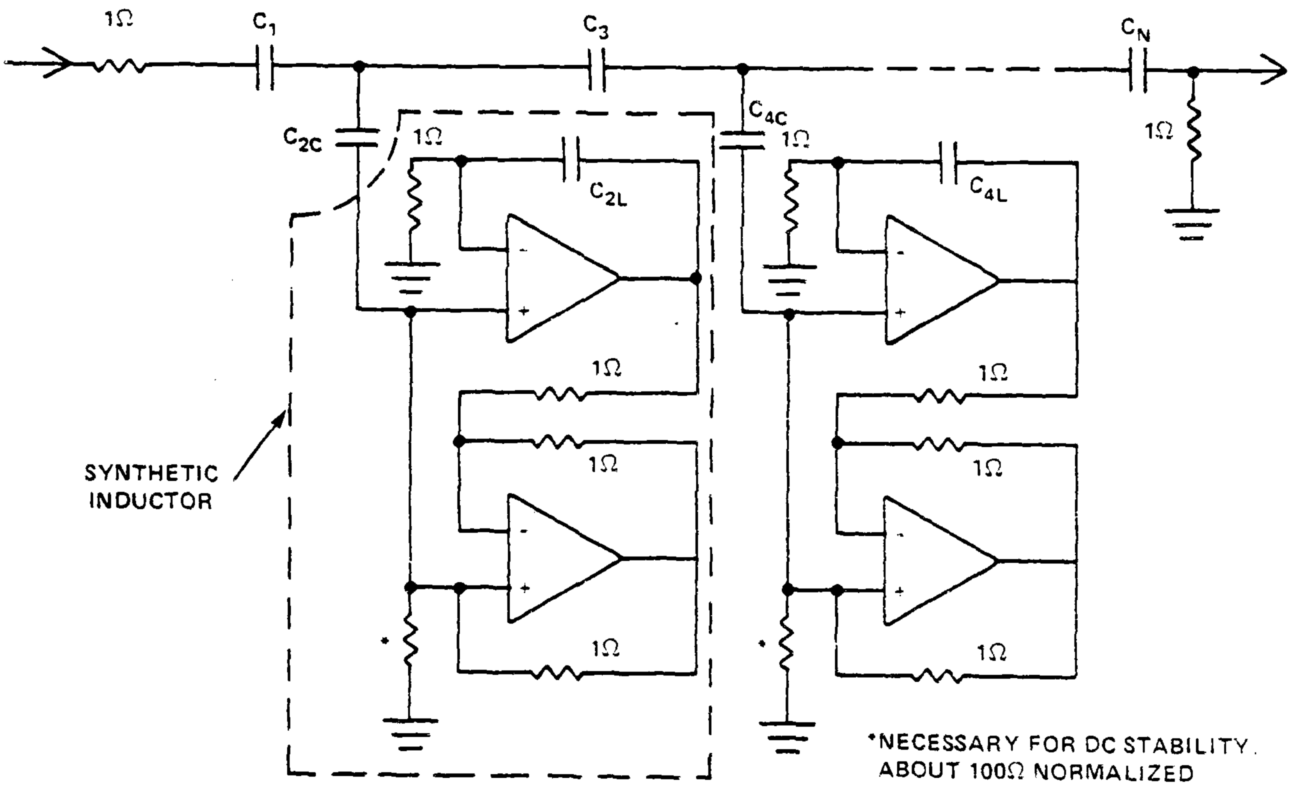


B. ACTIVE LOW-PASS

FIGURE 11. ELLIPTIC FILTER CIRCUITS



C. PASSIVE HIGH-PASS



D. ACTIVE HIGH-PASS

FIGURE 11. (CONT.)

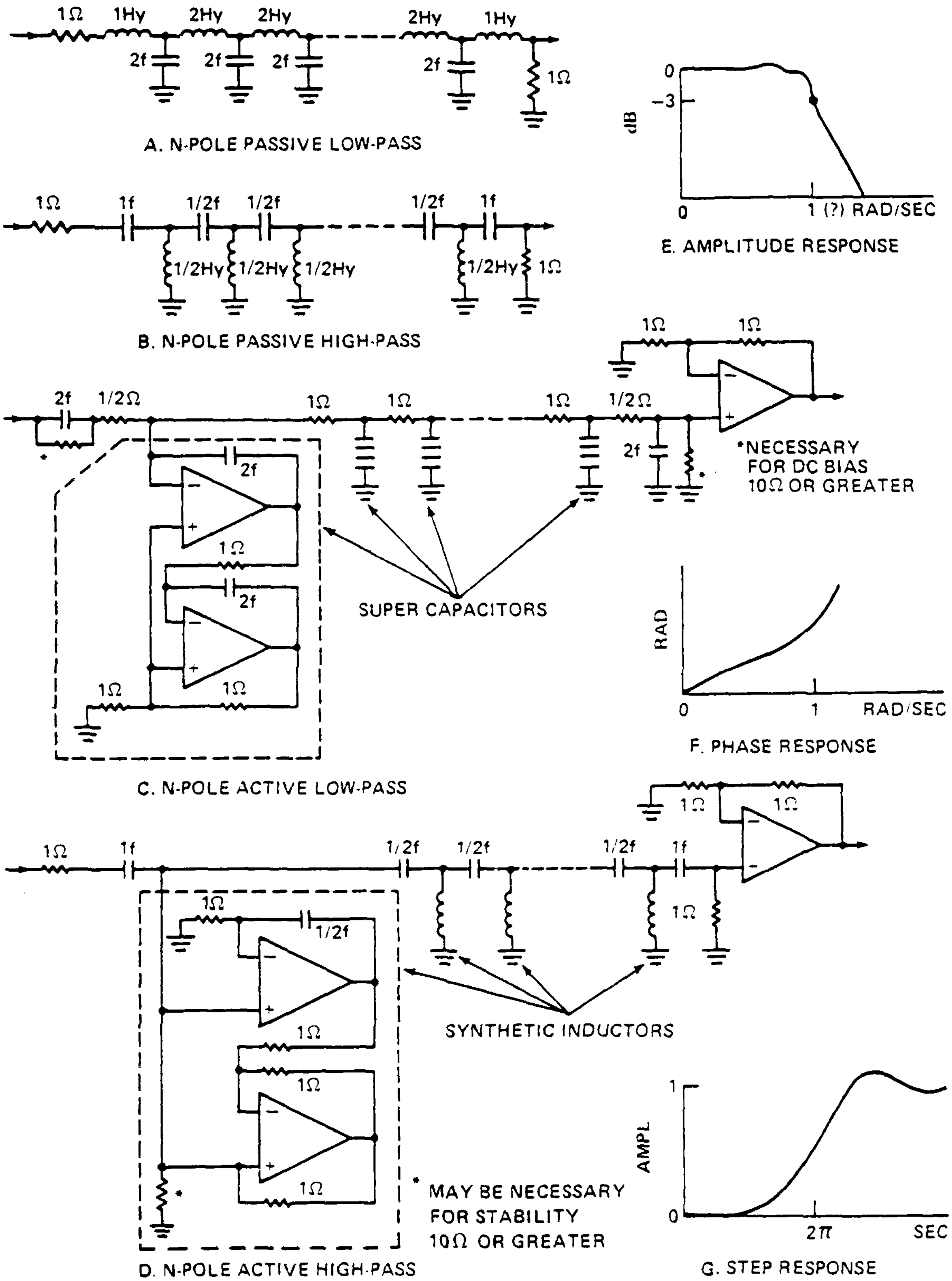


FIGURE 12. CONSTANT-K FILTERS

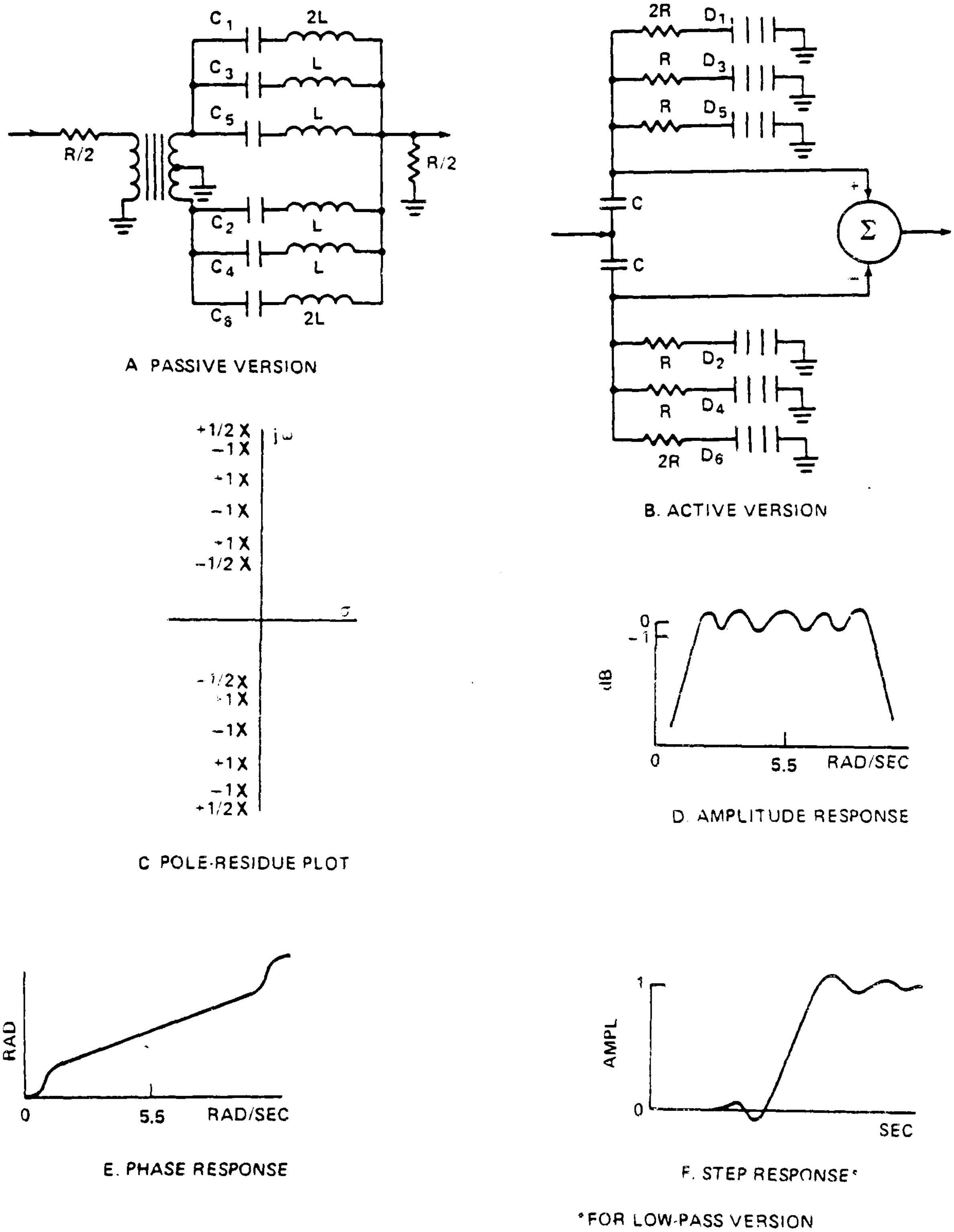


FIGURE 13. LERNER FILTER METHOD

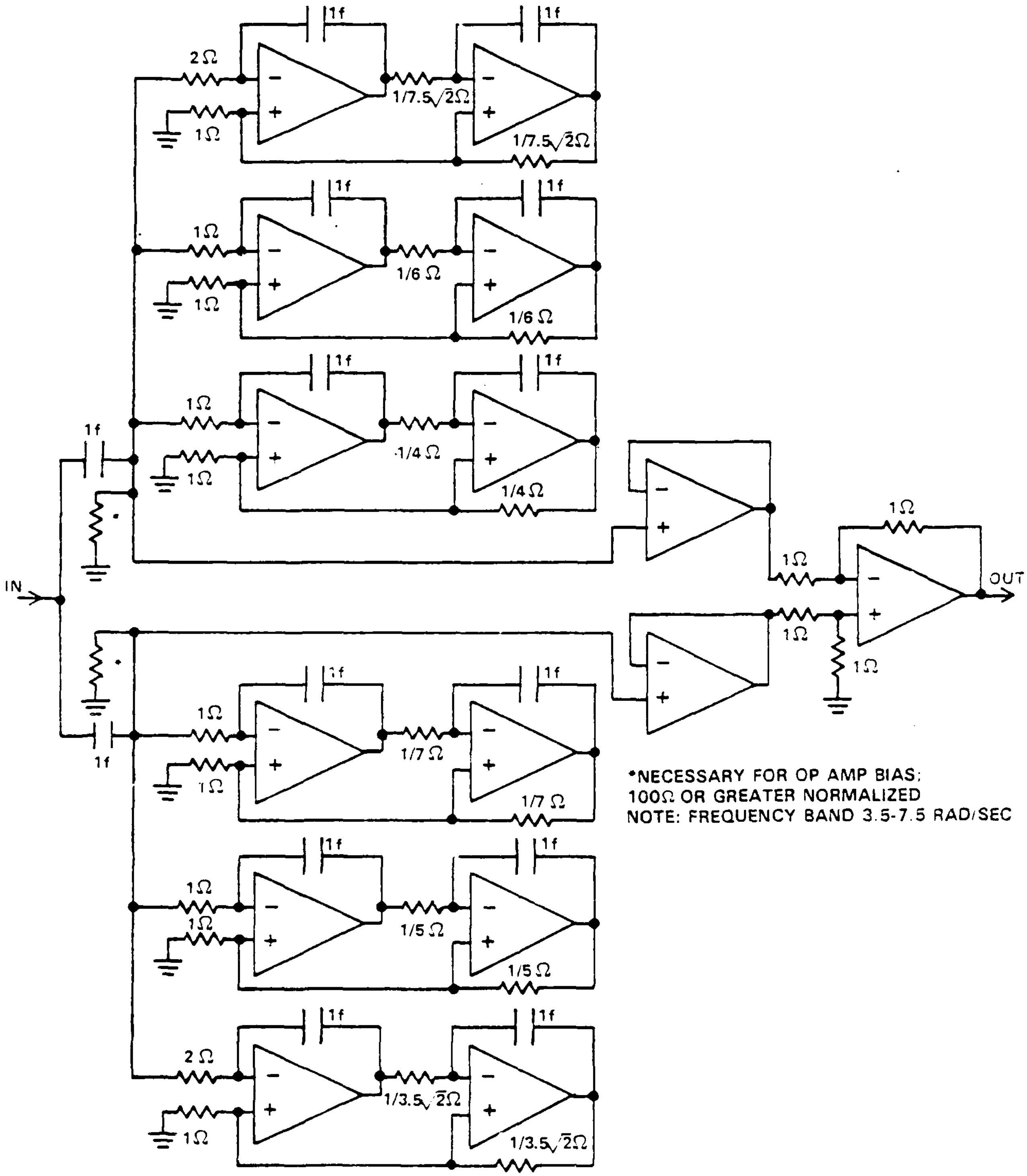


FIGURE 14. 12-POLE BANDPASS LERNER FILTER

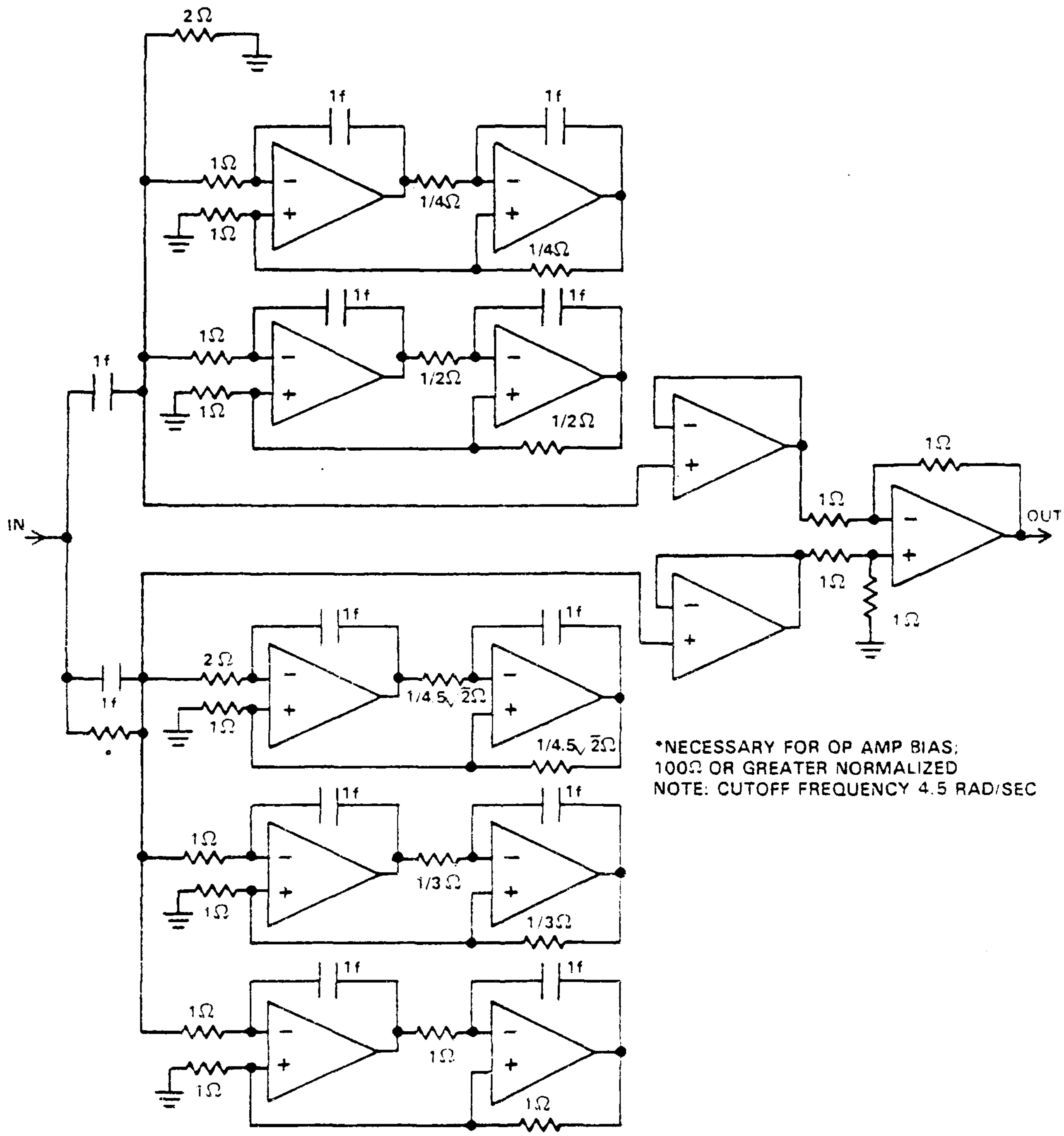


FIGURE 15. 11-POLE LOW-PASS LERNER FILTER

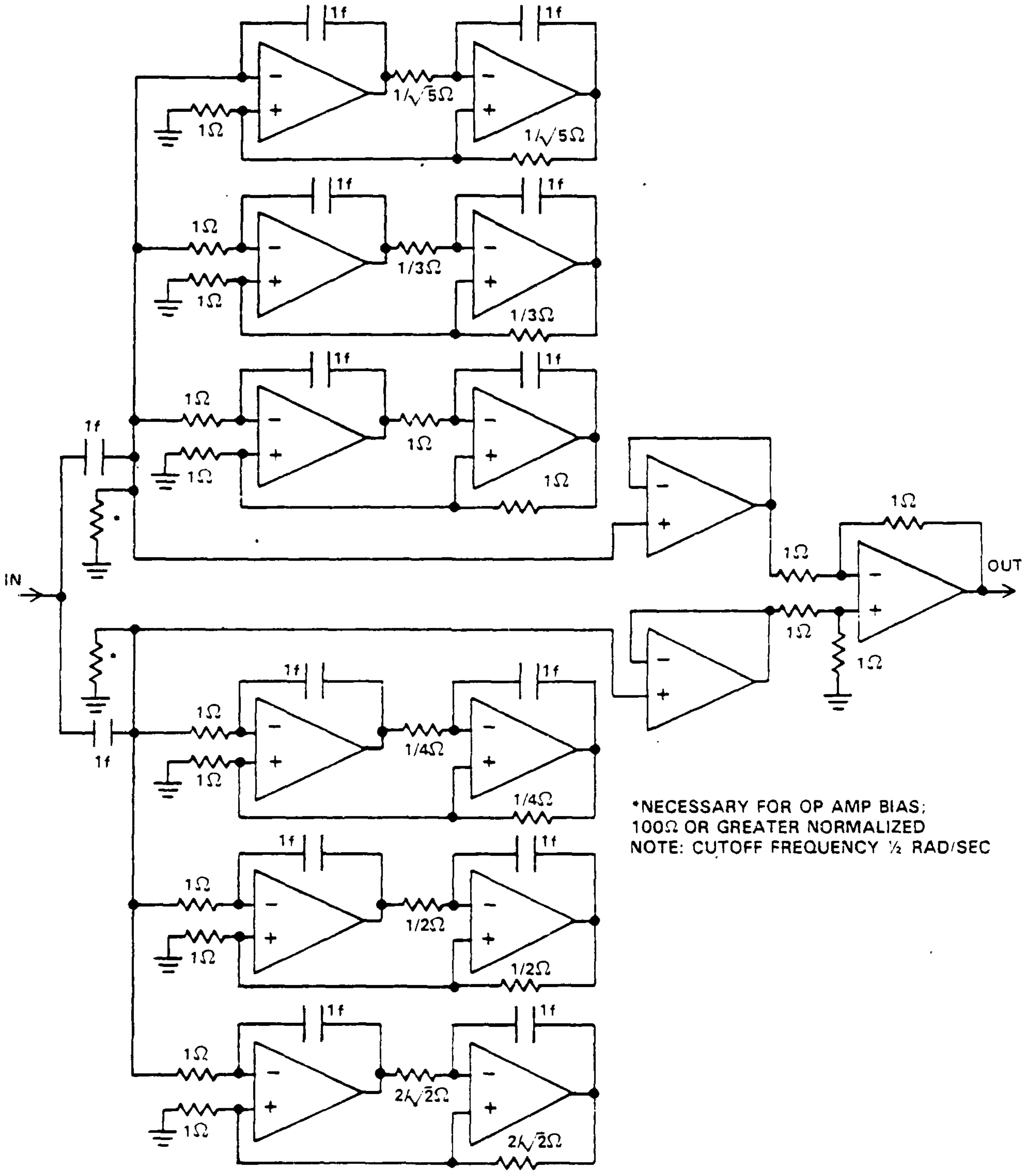
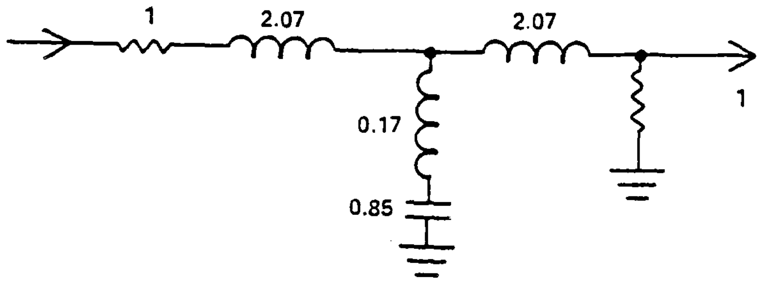
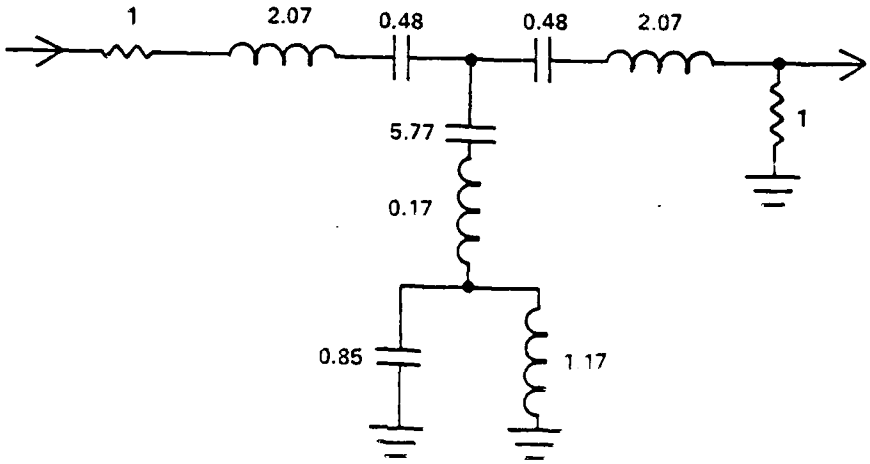


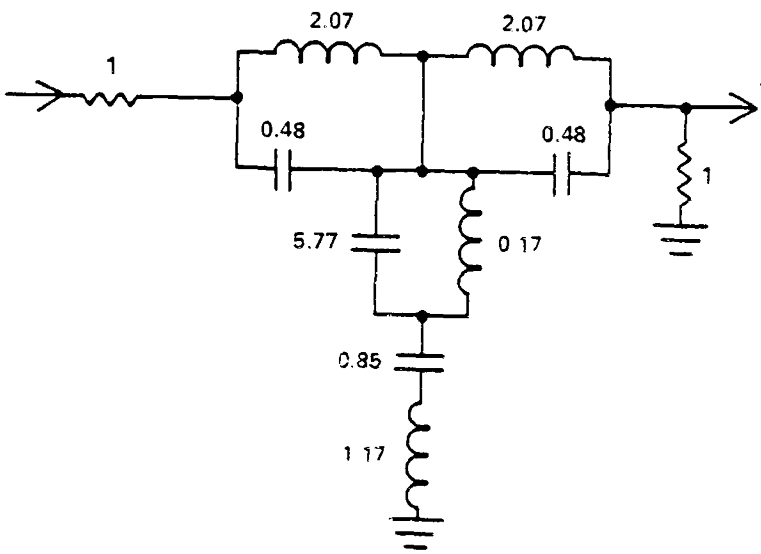
FIGURE 16. 11-POLE HIGH-PASS LERNER FILTER



A. PASSIVE LOW-PASS PROTOTYPE



B. PASSIVE BANDPASS PROTOTYPE



C. PASSIVE BANDSTOP PROTOTYPE

FIGURE 17 BANDPASS AND BANDSTOP TRANSFORMATIONS

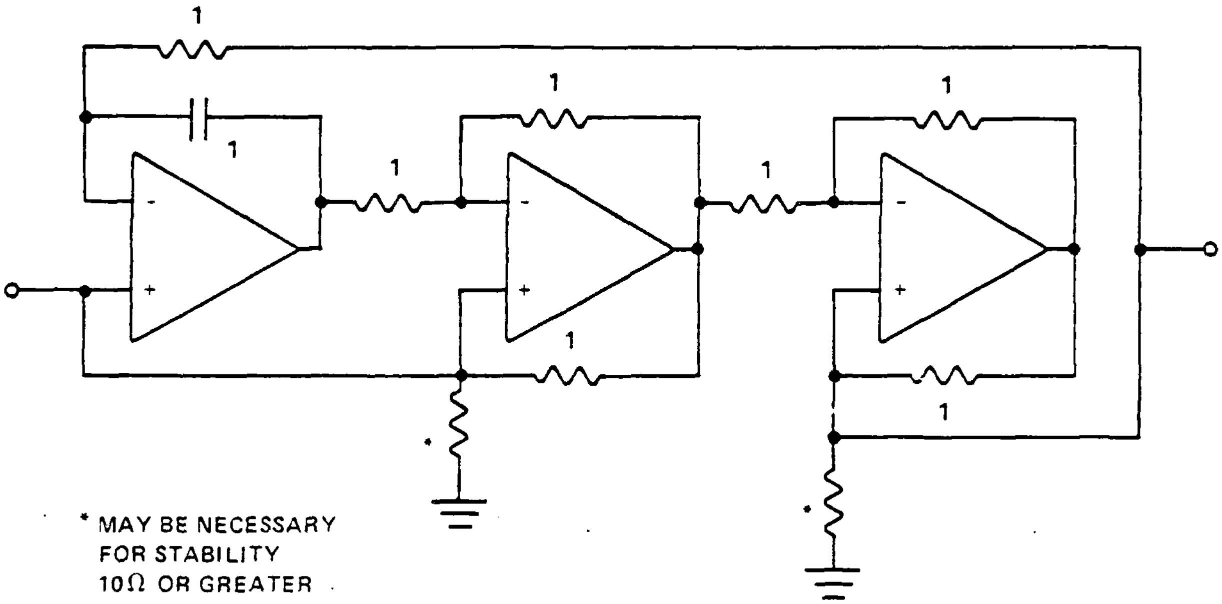
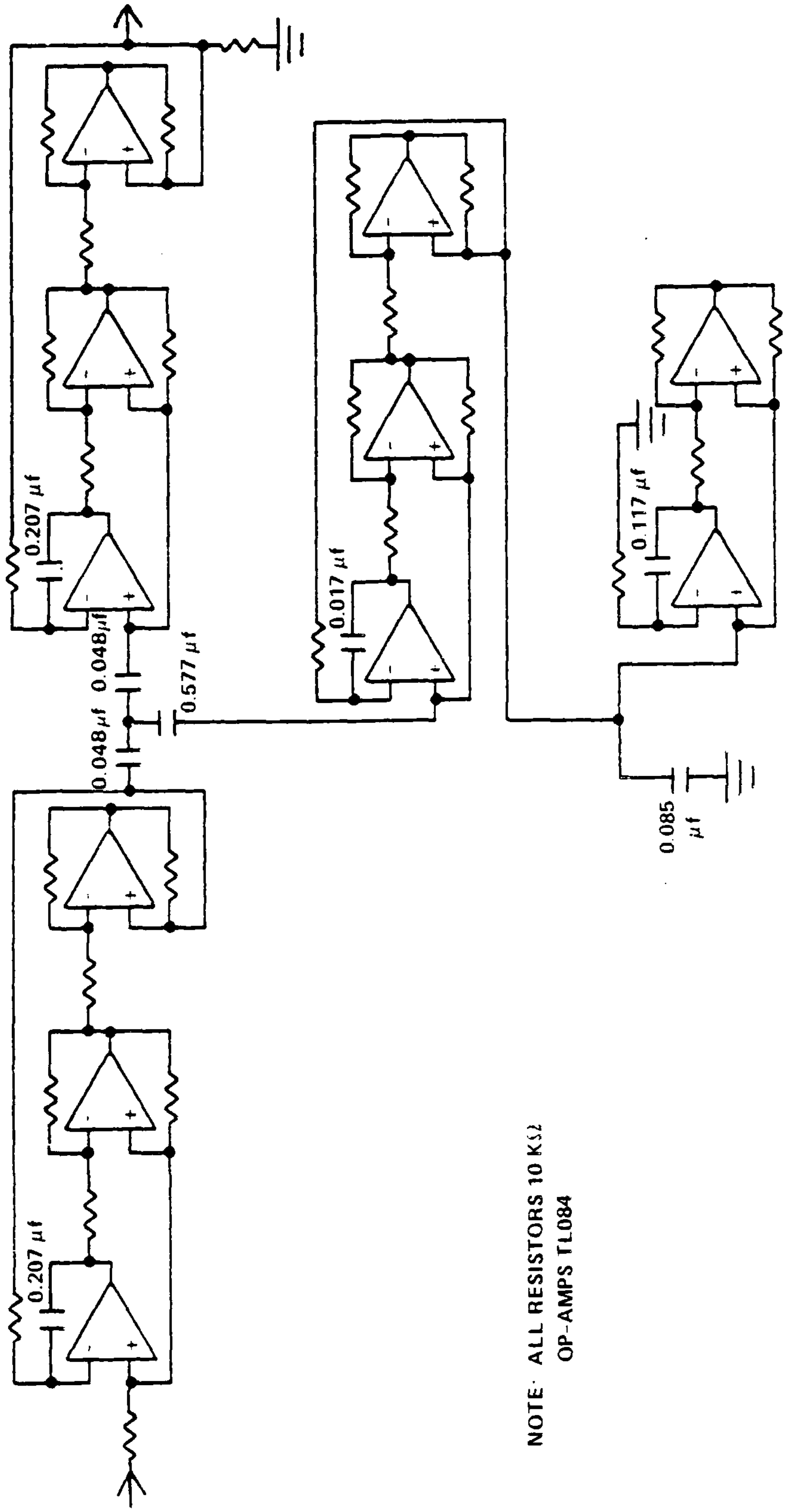


FIGURE 18. FLOATING SYNTHETIC INDUCTOR



NOTE: ALL RESISTORS 10 KΩ
OP-AMPS TL084

FIGURE 19 ACTIVE BANDPASS CIRCUIT

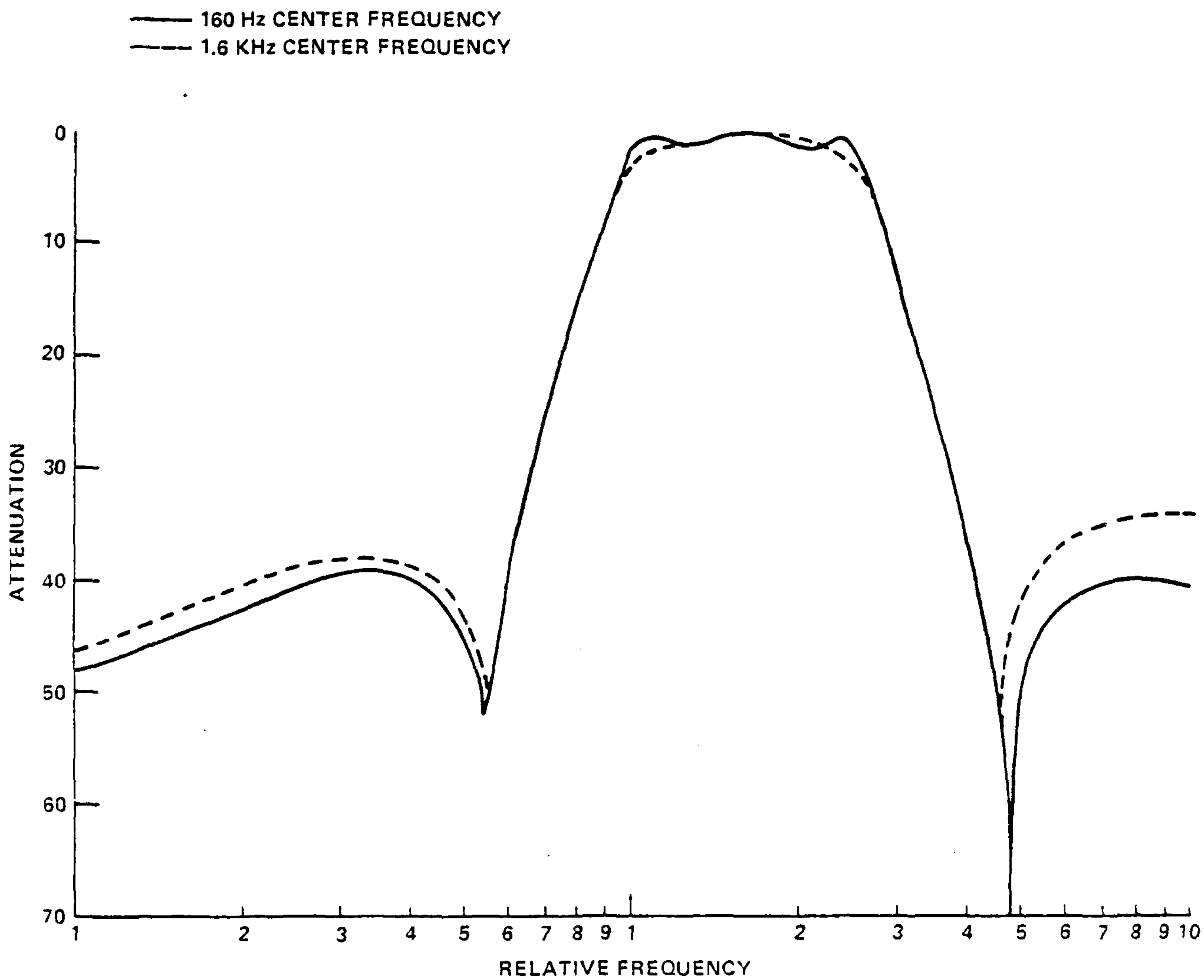


FIGURE 20. BANDPASS FILTER PERFORMANCE

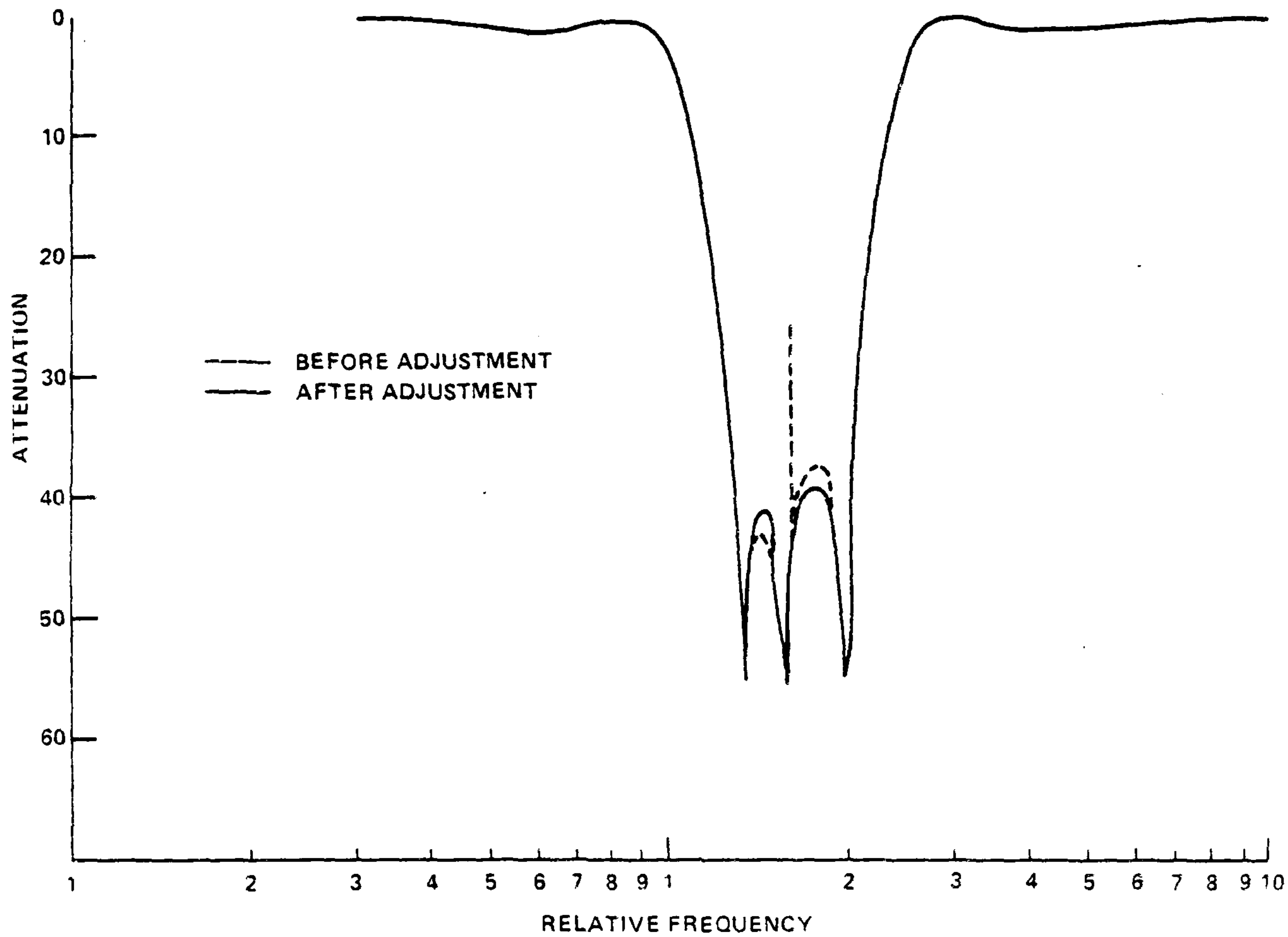
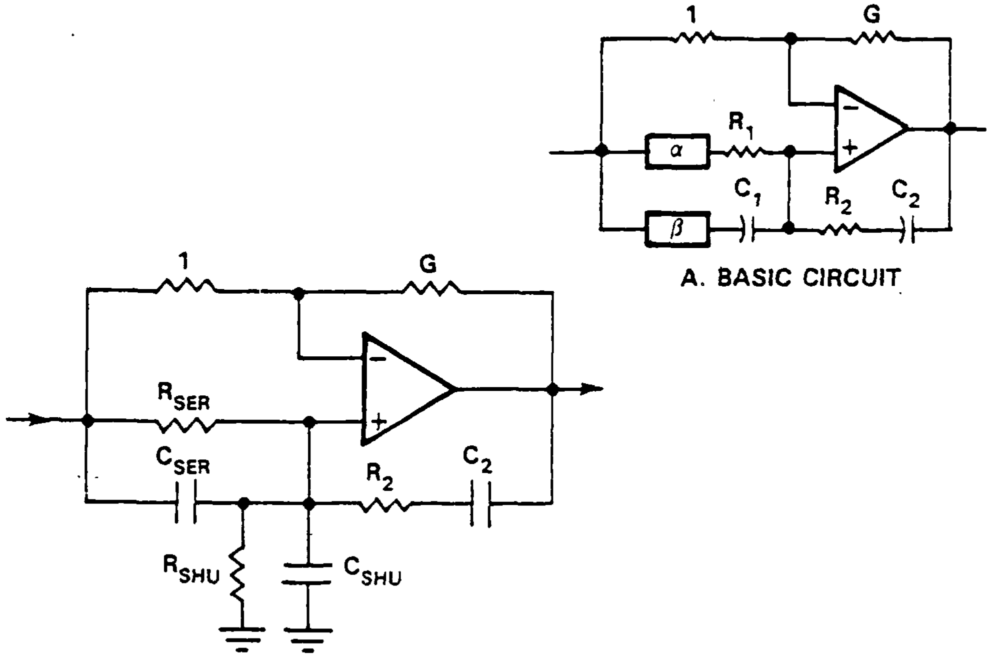


FIGURE 21. BANDSTOP FILTER PERFORMANCE



$$H(s) = K \frac{S^2 + D}{S^2 + AS + B}$$

A. BASIC CIRCUIT

B. COMPLETE CIRCUIT

COMPONENT VALUES

HIGHPASS	LOWPASS
LET $\beta = 1; C_1 = C_2 = 1$	LET $\alpha = 1, R_1 = R_2 = 1$
$R_1 = \dfrac{(1 - \frac{D}{B})}{A}$	$C_2 = \dfrac{(1 - \frac{B}{D})}{A}$
$R_2 = \dfrac{1}{BR_1}$	$C_1 = \dfrac{1}{BC_2}$
$G = \dfrac{(R_1 + R_2 - \frac{A}{B})}{R_1}$	$G = \dfrac{(C_1 + C_2 - \frac{A}{B})}{C_2}$
$\alpha = \dfrac{G + \frac{D}{B}}{G + 1}$	$\beta = \dfrac{G + \frac{B}{D}}{G + 1}$
$R_{SERIES} = \dfrac{R_1}{\alpha}$	$C_{SERIES} = \beta C_1$
$R_{SHUNT} = \dfrac{R_1}{(1 - \alpha)}$	$C_{SHUNT} = (1 - \beta) C_1$

FIGURE 22. SINGLE-OP-AMP RESONATOR WITH ZEROS

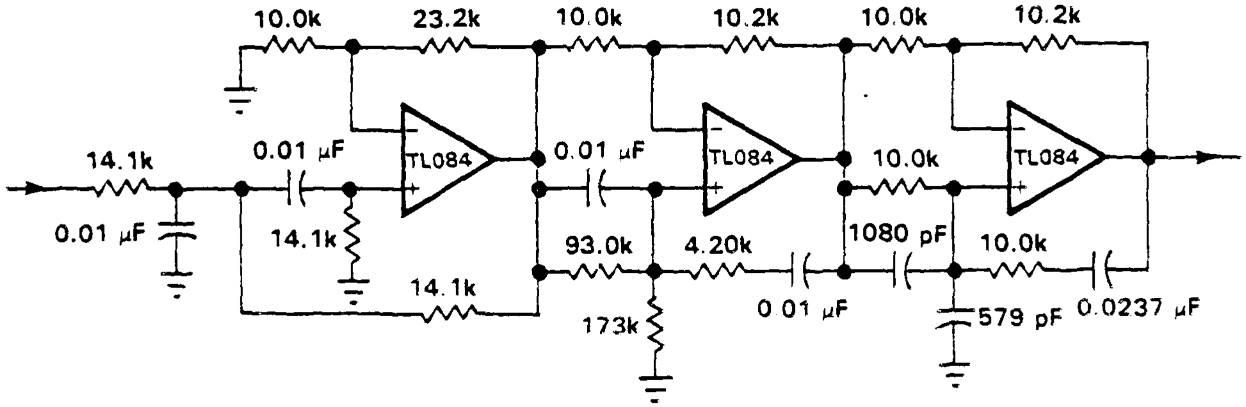


FIGURE 23. ELLIPTIC BANDPASS WITH SINGLE-OP-AMP SECTIONS

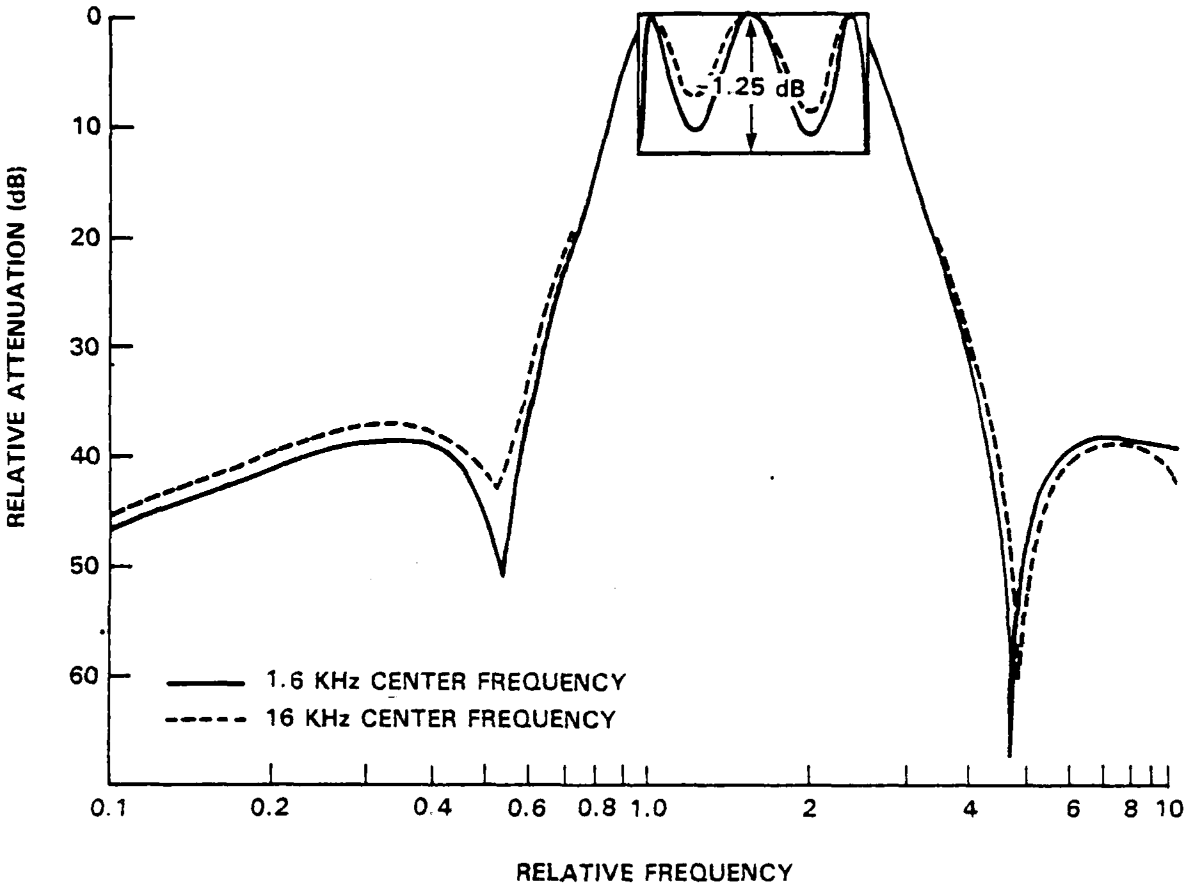
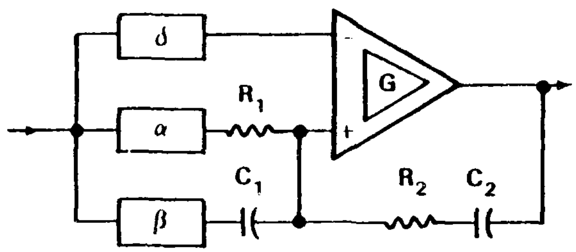
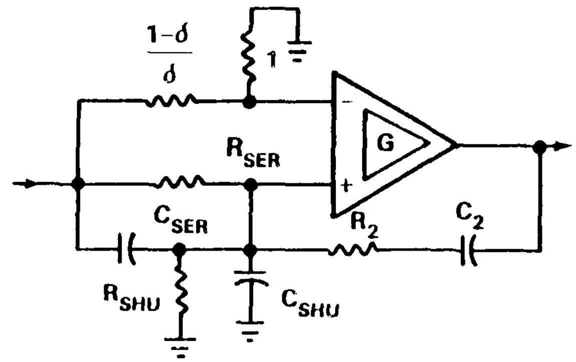


FIGURE 24. PERFORMANCE OF ELLIPTIC BANDPASS WITH SINGLE-OP-AMP SECTIONS



A. BASIC CIRCUIT

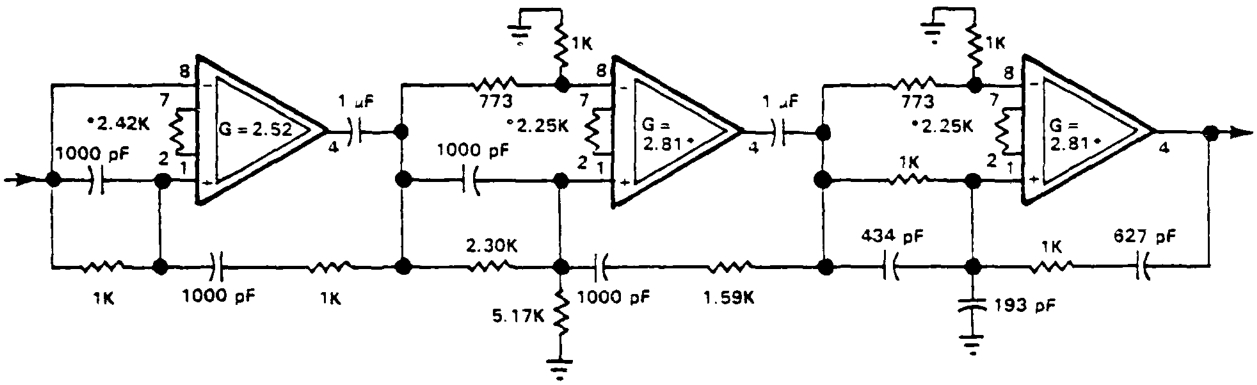
$$H(s) = K \frac{s^2 + D}{s^2 + AS + B}$$



FILTER FUNCTION	FORMULA	δ	G	R_{SER}	R_2	R_{SHU}	C_{SER}	C_2	C_{SHU}
POLE-ZERO QUAD, LOWPASS WITH IMAGINARY ZEROS	$\frac{s^2+D}{s^2+As+B}$ D>B	$1 - \frac{1}{2+B/D}$	$3 - \frac{A}{\sqrt{B}}$	1	1	NONE	$\frac{1}{\sqrt{B}}\left(2 - \frac{3}{2+B/D}\right)$	$\frac{1}{\sqrt{B}}$	$\frac{1}{\sqrt{B}}\left(\frac{3}{2+B/D} - 1\right)$
POLE-ZERO QUAD HIGHPASS WITH IMAGINARY ZEROS	$\frac{s^2+D}{s^2+As+B}$ D<B	$1 - \frac{1}{2+D/B}$	$3 - \frac{A}{\sqrt{B}}$	$\frac{1/\sqrt{B}}{2 - \frac{3}{2+D/B}}$	$\frac{1}{\sqrt{B}}$	$\frac{1/\sqrt{B}}{\frac{3}{2+D/B} - 1}$	1	1	NONE
POLE ZERO QUAD NOTCH	$\frac{s^2+B}{s^2+As+B}$	2/3	$3 - \frac{A}{\sqrt{B}}$	$\frac{1}{\sqrt{B}}$	$\frac{1}{\sqrt{B}}$	NONE	1	1	NONE
POLE ZERO QUAD, ALLPASS	$\frac{s^2-As+B}{s^2+As+B}$	$1 - \frac{1}{3+A/\sqrt{B}}$	$3 - \frac{A}{\sqrt{B}}$	$\frac{1}{\sqrt{B}}$	$\frac{1}{\sqrt{B}}$	NONE	1	1	NONE
POLE-PAIR SINGLE ZERO, BANDPASS	$\frac{s}{s^2+As+B}$	1	$3 - \frac{A}{\sqrt{B}}$	$\frac{1}{\sqrt{B}}$	$\frac{1}{\sqrt{B}}$	NONE	1	1	NONE
POLE-ZERO PAIR, ALLPASS	$\frac{s-1}{s+1}$	1/2	DOESN'T MATTER	1	NONE	NONE	NONE	NONE	1

B. COMPLETE CIRCUIT

FIGURE 25. RESONATOR WITH ZEROS WITH NO OP-AMP



NOTES:
1. AMPLIFIERS: TL592
*NOMINAL VALUE; ADJUSTED

FIGURE 26. ELLIPTIC BANDPASS WITHOUT OP-AMPS

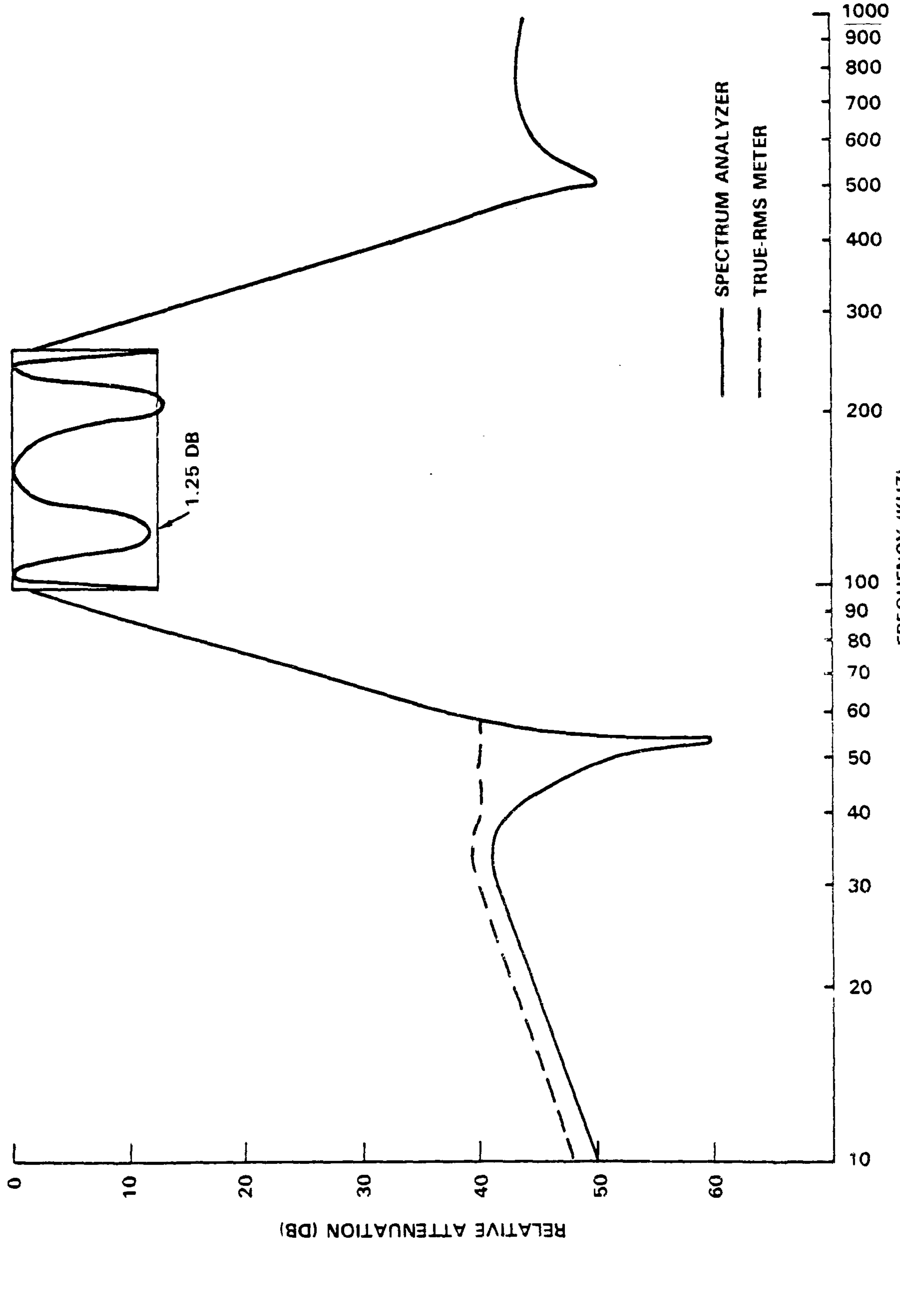


FIGURE 27. PERFORMANCE OF ELLIPTIC BANDPASS WITHOUT OF-AMPS

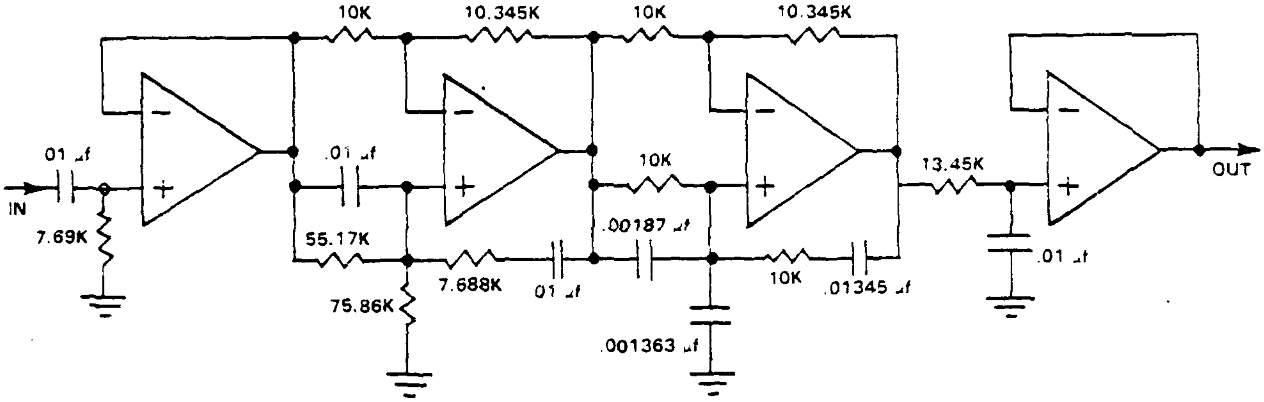


FIGURE 28. QUASI-ELLIPTIC BANDPASS CIRCUIT

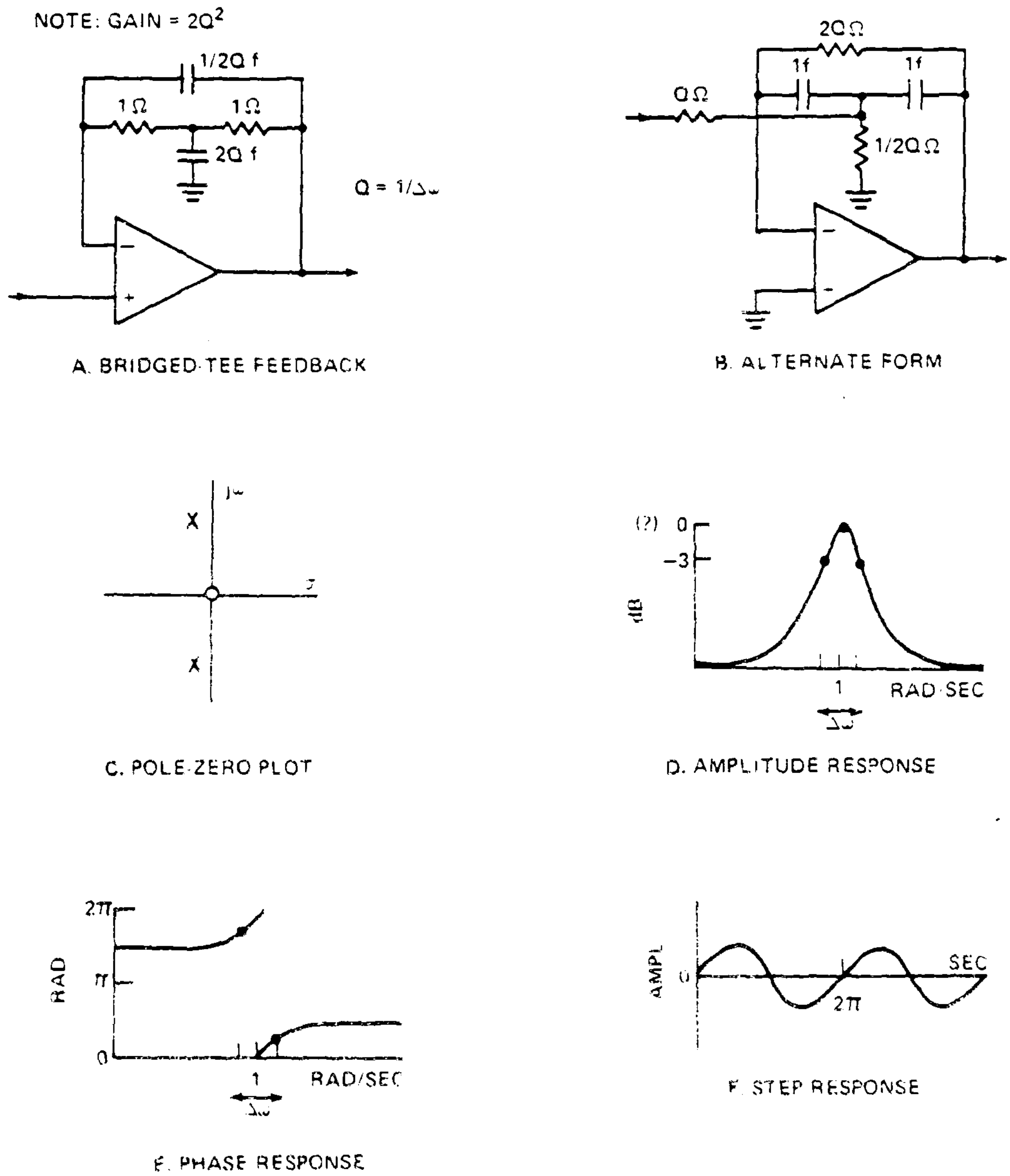
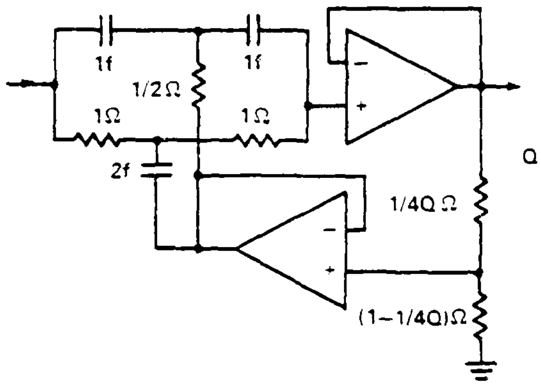
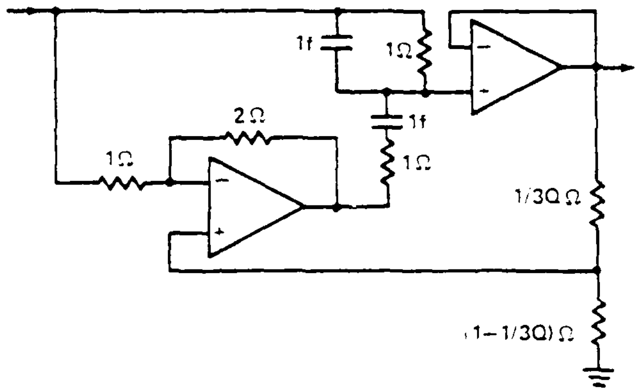


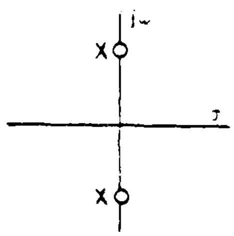
FIGURE 29. NARROWBAND FILTERS



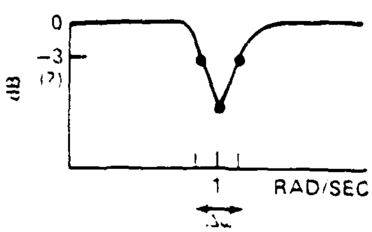
A. TWIN-TEE WITH FEEDBACK



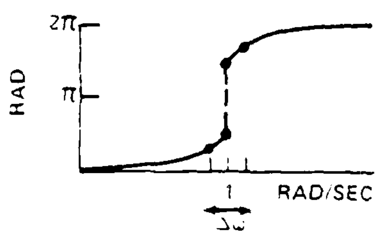
B. WIEN BRIDGE WITH FEEDBACK



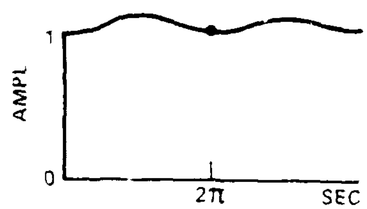
C. POLE-ZERO PLOT



D. AMPLITUDE RESPONSE



E. PHASE RESPONSE



F. STEP RESPONSE

FIGURE 30. NOTCH FILTERS

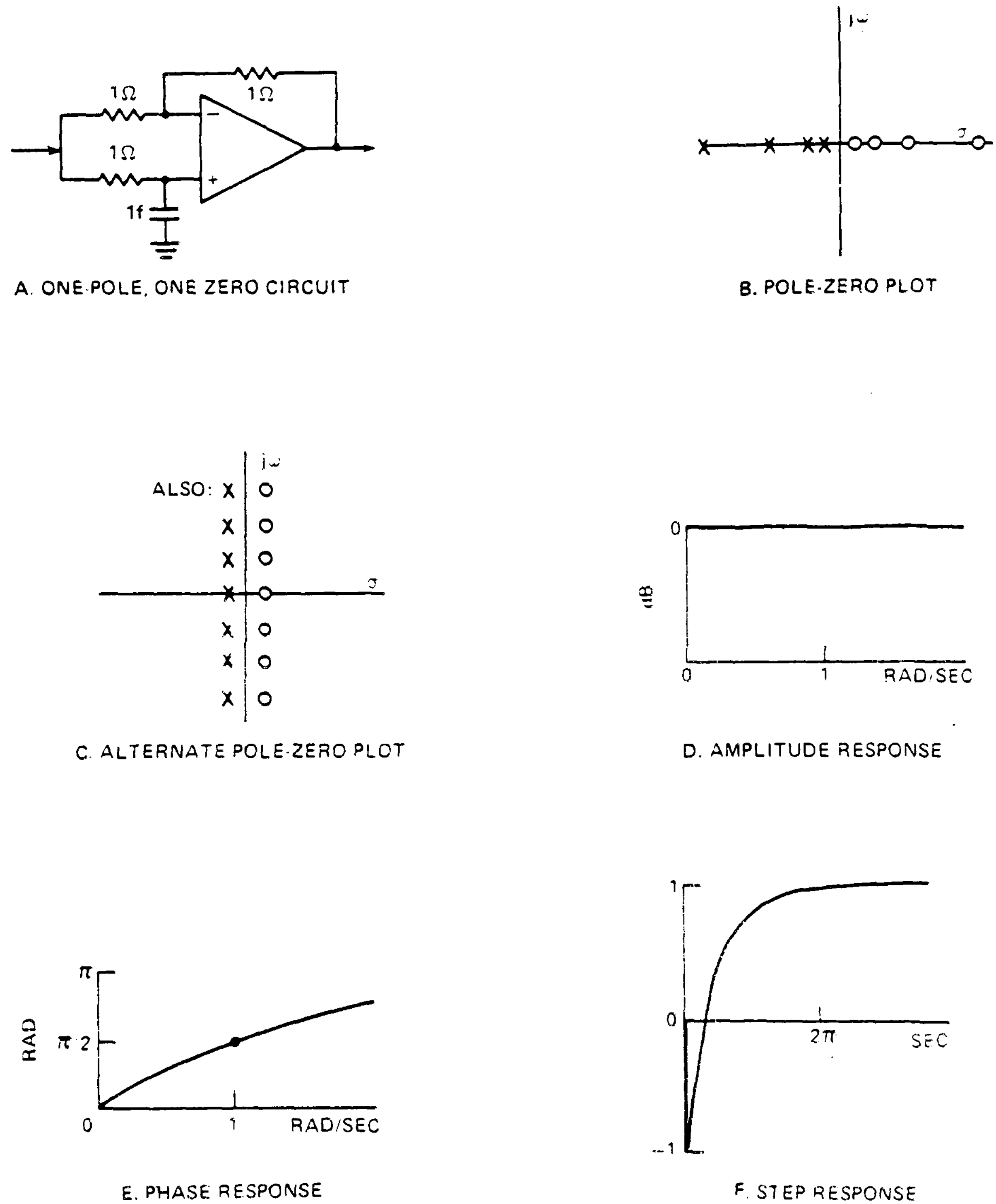


FIGURE 31. ALL-PASS FILTERS

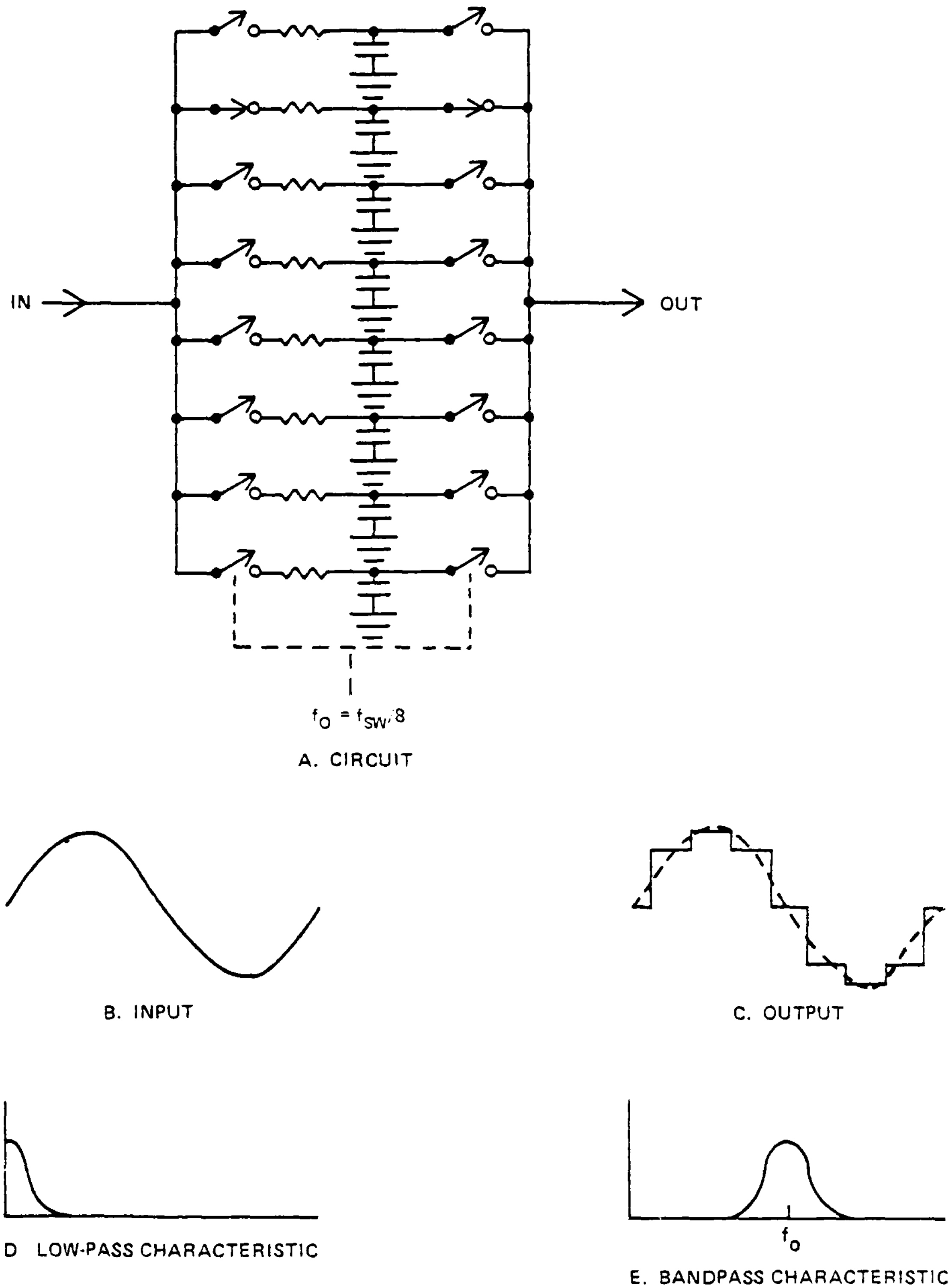
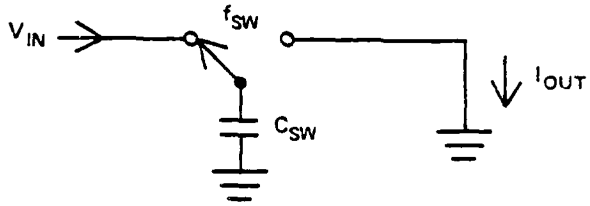
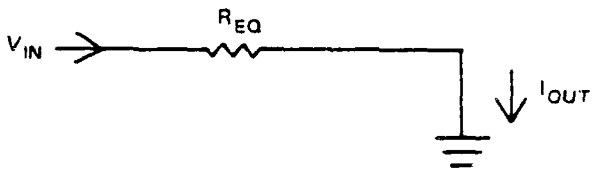


FIGURE 32. COMMUTATING FILTER

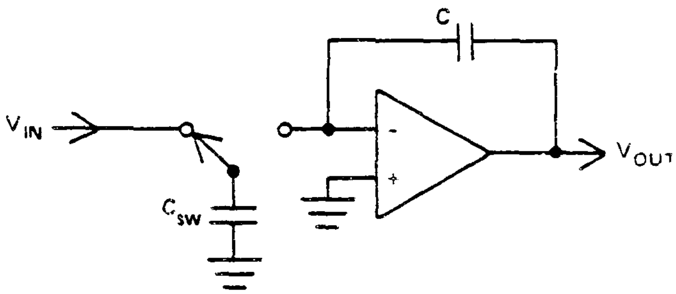


A. CIRCUIT



$$R_{EQ} = \frac{1}{C_{SW} f_{SW}}$$

B EQUIVALENT CIRCUIT



$$R_{EQ} = \frac{1}{f_{SW}} \frac{C}{C_{SW}}$$

C. SWITCHED-CAPACITOR INTEGRATOR

FIGURE 33. SWITCHED-CAPACITOR FILTER METHOD

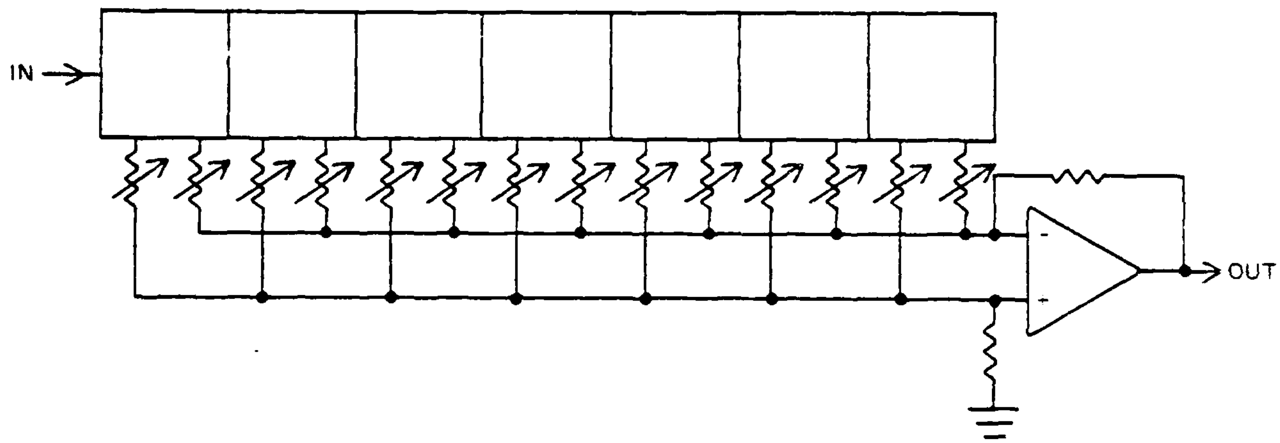


FIGURE 34. TRANSVERSAL FILTER

TABLE 1. RECOMMENDED SOURCES

1. Williams, A. B., Active Filter Design, Artech House, 1975.

2. Hilburn and Johnson, Manual of Active Filter Design, McGraw-Hill Book Co., Inc., New York, NY, 1973.

3. Blinchikoff and Zverev, Filtering in Time and Frequency Domain, John Wiley & Sons, Inc., New York, NY, 1976.

4. Tobey, Graeme and Huelsman, Operational Amplifiers, Design and Applications, McGraw-Hill Book Co., Inc., New York, NY, 1971.

5. Delagrange, A. D., An Operational Amplifier Primer, NOL TR 72-166, 14 Aug 1972.

6. Delagrange, A. D., A Useful Filter Family, NSWC WOL TR 75-170, 20 Oct 1975.

7. Guillemin, E. A., Synthesis of Passive Networks, John Wiley & Sons, Inc., New York, NY, 1957.

8. Lam, H. Y-F., Analog and Digital Filters; Design and Realization, Prentice-Hall, 1979.

9. Lancaster, D., Active Filter Cookbook, Howard W. Sams, Inc., 1975.

10. Huelsman and Allen, Introduction to the Theory and Design of Active Filters, McGraw-Hill Book Co., Inc., New York, NY, 1980.

TABLE 2. COMPARISON OF FILTER METHODS

Property \ Method	Passive	Active/Analog	Digital
Uses Inductors?	Yes	No	No
Requires Power Supply?	No	Yes	Yes
High-Frequency Capability	Good	Fair	Poor
Low-Frequency Capability	Poor	Fair	Good
Input Impedance	Matched	High	Irrelevant
Output Impedance	Matched	Low	Irrelevant
Number of Sections	Many	Few	Expandable
Dynamic Range	Good	Fair	Expandable
Expense	Medium	Low	High
Match to Calculations	Poor	Good	Exact

TABLE 3. APPROXIMATE DECIBLE CONVERSON

Number of Decibels	Equivalent Gain	Equivalent Attenuation
0	1	1
1	1.1	0.9
3	1.4	0.7
6	2	0.5
10	3	0.3
20	10	0.1

TABLE 4. BUTTERWORTH VALUES

Order	G_1	G_2	G_3	G_4	NOISE BANDWIDTH
1					1.571
2	1.5858				1.110
3	2.0000				1.047
4	1.1522	2.2346			1.026
5	1.3820	2.3820			1.017
6	1.0681	1.5858	2.4824		1.012
7	1.1981	1.7530	2.5550		1.008
8	1.0385	1.3371	1.8889	2.6098	1.006
9	1.1206	1.4679	2.0000	2.6527	1.005

NOTE: ALL RESISTORS 1.000 EXCEPT GAIN-SETTING

TABLE 5. CHEBYSHEV 0.1 dB RIPPLE VALUES

LOW-PASS	R_0	G_1	R_1	G_2	R_2	G_3	R_3	G_4	R_4	3dB BANDWIDTH	NOISE BANDWIDTH
1	0.1526									6.6039	10.292
2		1.6968	0.5493							1.9432	2.144
3	1.0316	2.2542	0.7693							1.3890	1.442
4		1.3839	1.2670	2.5419	0.8671					1.2131	1.233
5	1.8556	1.9065	1.2540	2.6953	0.9148					1.1347	1.142
6		1.3318	1.9486	2.2490	1.1983	2.7842	0.9410			1.0929	1.094
7	2.6541	1.8185	1.7402	2.4586	1.1522	2.8396	0.9568			1.0680	1.065
8		1.3142	2.6206	2.1547	1.5500	2.5923	1.1188	2.8763	0.9670	1.0519	1.047
9	3.4428	1.7834	2.2286	2.3691	1.4177	2.6820	1.0948	2.9018	0.9739	1.0410	1.034
HIGH-PASS	$1/R_0$	G_1	$1/R_1$	G_2	$1/R_2$	G_3	$1/R_3$	G_4	$1/R_4$		

TABLE 6. CHEBYSHEV 0.5 dB RIPPLE VALUES

LOW-PASS	R_0	G_1	R_1	G_2	R_2	G_3	R_3	G_4	R_4	3dB BANDWIDTH	NOISE BANDWIDTH
1	0.3493									2.8629	4.498
2		1.8422	0.8121							1.3897	1.489
3	1.6014	2.4139	0.9356							1.1675	1.167
4		1.5818	1.6750	2.6599	0.9697					1.0931	1.066
5	2.7600	2.1510	1.4483	2.7800	0.9826					1.0593	1.021
6		1.5372	2.5238	2.4476	1.3019	2.8465	0.9887			1.0410	0.997
7	3.9037	2.0839	1.9847	2.6117	1.2155	2.8869	0.9920			1.0310	0.983
8		1.5221	3.3700	2.3792	1.6698	2.7115	1.1614	2.9138	0.9941	1.0230	0.974
9	5.0402	2.0570	2.5291	2.5481	1.4867	2.7767	1.1255	2.9314	0.9954	1.0182	0.967
HIGH-PASS	$1/R_0$	G_1	$1/R_1$	G_2	$1/R_2$	G_3	$1/R_3$	G_4	$1/R_4$		

TABLE 7. CHEBYSHEV 1 dB RIBBLE VALUES

LOW-PASS	R_0	G_1	R_1	G_2	R_2	G_3	R_3	G_4	R_4	3dB BANDWIDTH	NOISE BANDWIDTH
1	0.5088									1.9654	3.088
2		1.9545	0.9524							1.2176	1.253
3	2.0236	2.5044	1.0029							1.0949	1.041
4		1.7254	1.8919	2.7190	1.0068					1.0530	0.973
5	3.4543	2.2851	1.5262	2.8200	1.0059					1.0338	0.943
6		1.6857	2.8317	2.5450	1.3390	2.8751	1.0047			1.0234	0.927
7	4.8682	2.2290	2.0831	2.6831	1.7371	2.9082	1.0037			1.0172	0.918
8		1.6720	3.7726	2.4889	1.7128	2.7656	1.1756	2.9298	1.0029	1.0132	0.911
9	6.2763	2.2064	2.6503	2.6314	1.5106	2.8191	1.1356	2.9445	1.0024	1.0104	0.907
HIGH-PASS	$1/R_0$	G_1	$1/R_1$	G_2	$1/R_2$	G_3	$1/R_3$	G_4	$1/R_4$		

TABLE 8. CHEBYSHEV 3 dB RIPPLE VALUES

LOW-PASS	R_0	G_1	R_1	G_2	R_2	G_3	R_3	G_4	R_4	NOISE BANDWIDTH
1	0.9976									1.571
2		2.2335	1.1885							0.864
3	3.3487	2.6740	1.0916							0.774
4		1.9718	2.5002	2.8268	1.0523					0.744
5	5.6329	2.5322	1.6286	2.8866	1.0336					0.731
6		2.0425	3.3557	2.7108	1.3843	2.9218	1.0234			0.724
7	7.9061	2.4957	2.2127	2.8009	1.2626	2.9427	1.0172			0.720
8		2.0326	4.4591	2.6754	1.7651	2.8535	1.1922	2.9563	1.0132	0.717
9	10.1756	2.4810	2.8101	2.7685	1.5379	2.8872	1.1473	2.9655	1.0104	0.715
HIGH-PASS	$1/R_0$	G_1	$1/R_1$	G_2	$1/R_2$	G_3	$1/R_3$	G_4	$1/R_4$	

TABLE 9 BESSEL VALUES

LOW-PASS	R_0	G_1	R_1	G_2	R_2	G_3	R_3	G_4	R_4	NOISE BANDWIDTH
1	1.0000									1.571
2		1.2679	0.7852							1.152
3	0.7536	1.5529	0.6885							1.073
4		1.0842	0.7046	1.7585	0.6284					1.047
5	0.6636	1.2255	0.6406	1.9089	0.5579					1.038
6		1.0405	0.6227	1.3639	0.5913	2.0228	0.5244			1.039
7	0.5936	1.1216	0.5823	1.4857	0.5464	2.1121	0.4877			1.042
8		1.0238	0.5606	1.2130	0.5441	1.5932	0.5104	2.1842	0.4555	1.046
9	0.5384	1.0758	0.5371	1.3012	0.5135	1.6852	0.4804	2.4278	0.4304	1.049

TABLE 10. ELLIPTIC 0.28 dB RIPPLE 3-POLE 2-ZERO VALUES

PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
		STOPBAND/PASSBAND RATIO		1.15	1.52	2.28	5.24
		STOPBAND ATTENUATION		8.8	19.9	32.5	55.2
L_1	R_1	$1/C_1$	$1/C_1$	0.7944	1.0813	1.2382	1.3260
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	1.4111	0.4251	0.1469	0.0246
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	0.4575	0.8099	1.0067	1.1172
L_3	R_3	$1/C_3$	$1/C_3$	0.7944	1.0813	1.2382	1.3260

TABLE 11. ELLIPTIC 0.28 dB RIPPLE 5-POLE 4-ZERO VALUES

PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
			STOPBAND/PASSBAND RATIO	1.15	1.52	2.28	5.24
			STOPBAND ATTENUATION	29.5	48.9	70.0	107.9
L_1	R_1	$1/C_1$	$1/C_1$	1.1677	1.3332	1.4084	1.4475
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	0.3872	0.1511	0.0567	0.0098
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	0.9858	1.1701	1.2534	1.2972
L_3	R_3	$1/C_3$	$1/C_3$	1.5108	1.9056	2.1283	2.2553
L_4	R_{4L}	$1/C_4$	$1/C_{4C}$	1.3083	0.4257	0.1519	0.0258
C_4	R_{4C}	$1/L_4$	$1/C_{4L}$	0.5458	0.9370	1.1553	1.2792
L_5	R_5	$1/C_5$	$1/C_5$	0.7337	1.1225	1.3216	1.4318

TABLE 12. ELLIPTIC 0.28 dB RIPPLE 7-POLE 6-ZERO VALUES

PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
			STOPBAND/PASSBAND RATIO	1.15	1.52	2.28	5.24
			STOPBAND ATTENUATION	50.9	78.0	107.5	160.6
L_1	R_1	$1/C_1$	$1/C_1$	1.3232	1.4204	1.4623	1.4836
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	0.2000	0.0795	0.0300	0.0052
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	1.1584	1.2666	1.3137	1.3378
L_3	R_3	$1/C_3$	$1/C_3$	1.6183	2.0239	2.2400	2.3612
L_4	R_{4L}	$1/C_4$	$1/C_{4C}$	1.0436	0.3814	0.1411	0.0243
C_4	R_{4C}	$1/L_4$	$1/C_{4L}$	0.7013	1.0861	1.3012	1.4241
L_5	R_5	$1/C_5$	$1/C_5$	1.3663	1.8941	2.1859	2.3514
L_6	R_{6L}	$1/C_6$	$1/C_{6C}$	0.7233	0.2673	0.0985	0.0169
C_6	R_{6C}	$1/L_6$	$1/C_{6L}$	0.7893	1.0886	1.2413	1.3248
L_7	R_7	$1/C_7$	$1/C_7$	0.9741	1.2588	1.3974	1.4720

TABLE 13. ELLIPTIC 1.25 dB RIPPLE 3-POLE 2-ZERO VALUES

PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
		STOPBAND/PASSBAND RATIO		1.15	1.52	2.28	5.24
		STOPBAND ATTENUATION		15.3	26.8	39.4	62.2
L_1	R_1	$1/C_1$	$1/C_1$	1.4922	1.8694	2.0701	2.1819
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	1.4198	0.4822	0.1732	0.0294
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	0.4547	0.7140	0.8537	0.9317
L_3	R_3	$1/C_3$	$1/C_3$	1.4922	1.8694	2.0701	2.1819

TABLE 14. ELLIPTIC 1.25 dB RIPPLE 5-POLE 4-ZERO VALUES

			STOPBAND/PASSBAND RATIO	1.15	1.52	2.28	5.24
			STOPBAND ATTENUATION	36.5	55.9	76.9	114.9
PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
L_1	R_1	$1/C_1$	$1/C_1$	1.9444	2.1590	2.2572	2.3086
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	0.4805	0.1898	0.0715	0.0124
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	0.7945	0.9318	0.9949	1.0280
L_3	R_3	$1/C_3$	$1/C_3$	2.1125	2.6821	2.9919	3.1664
L_4	R_{4L}	$1/C_4$	$1/C_{4C}$	1.4882	0.5206	0.1898	0.0325
C_4	R_{4C}	$1/L_4$	$1/C_{4L}$	0.4798	0.7662	0.9250	1.0152
L_5	R_5	$1/C_5$	$1/C_5$	1.4228	1.8974	2.1482	2.2887

TABLE 15. ELLIPTIC 1.25 dB RIPPLE 7-POLE 6-ZERO VALUES

			STOPBAND/PASSBAND RATIO	1.15	1.52	2.28	5.24
			STOPBAND ATTENUATION	57.8	85.0	114.4	167.6
PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
L_1	R_1	$1/C_1$	$1/C_1$	2.1357	2.2625	2.3175	2.3457
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	0.2535	0.1012	0.0382	0.0066
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	0.9141	0.9954	1.0310	1.0493
L_3	R_3	$1/C_3$	$1/C_3$	2.2594	2.8064	3.0977	3.2612
L_4	R_{4L}	$1/C_4$	$1/C_{4C}$	1.3297	0.4952	0.1842	0.0318
C_4	R_{4C}	$1/L_4$	$1/C_{4L}$	0.5504	0.8366	0.9966	1.0880
L_5	R_5	$1/C_5$	$1/C_5$	1.9028	2.6284	3.0243	3.2479
L_6	R_{6L}	$1/C_6$	$1/C_{6C}$	0.8835	0.3359	0.1250	0.0216
C_6	R_{6C}	$1/L_6$	$1/C_{6L}$	0.6462	0.8665	0.9786	1.0399
L_7	R_7	$1/C_7$	$1/C_7$	1.7004	2.0580	2.2350	2.3309

TABLE 16 COMPARISON OF FILTER TYPES

FILTER TYPE / PROPERTY	INDEPENDENT RC	BUTTERWORTH (MAXIMALLY FLAT)	CHEBYSHEV (TSCHEBYCHEFF) (EQUIRIPPLE)	ELLIPTIC (CAUER) (DOUBLE EQUIRIPPLE)	BESSEL (MAXIMALLY LINEAR PHASE)	CONSTANT - K IDENTICAL SECTIONS	LERNER
PASSBAND DROOP	TERRIBLE	FAIR	FAIR TO EXCELLENT	FAIR TO EXCELLENT	POOR	FAIR	GOOD
PASSBAND RIPPLE?	NO	NO	YES	YES	NO	YES	YES
TRANSITION STEEPNESS	POOR	FAIR	GOOD TO FAIR	EXCELLENT	POOR	GOOD	GOOD
STOPBAND MONOTONIC?	YES	YES	YES	NO	YES	YES	YES
ULTIMATE ATTENUATION	GOOD	GOOD	GOOD	POOR	GOOD	EXCELLENT	FAIR
ALL-POLE?	YES	YES	YES	NO	YES	YES	NO
NOISE BANDWIDTH	POOR	FAIR	FAIR	POOR TO GOOD	POOR	EXCELLENT	FAIR
MINIMUM PHASE?	YES	YES	YES	YES	YES	YES	NO
PHASE LINEARITY	FAIR	FAIR	POOR	POOR	GOOD	FAIR	EXCELLENT
TRANSIENT RESPONSE	EXCELLENT	FAIR	POOR	POOR	GOOD	FAIR	FAIR
COMPONENT SENSITIVITY	EXCELLENT	FAIR	POOR	POOR	GOOD	EXCELLENT	FAIR
EASILY EXPANDABLE?	YES	NO	NO	NO	NO	YES	IN A WAY
COMPLEXITY/ COST	EXCELLENT	GOOD	FAIR	POOR	GOOD	GOOD	FAIR
HIGH FREQ CAPABILITY*	EXCELLENT	GOOD	FAIR	POOR	GOOD	POOR	POOR

*ACTIVE VERSION

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APPENDIX A

FILTER RESPONSE DATA

The filter designs given in this series of reports have become too numerous to catalog all the responses, so only a sampling will be given. Circuits were built from the prototypes and tables given in this report, or in a few cases the listed references and the actual responses recorded.

The three curves normally sketched for each filter type are generated here--amplitude, phase and step response. The plots were made using a Hewlett-Packard* 3561A Dynamic Signal Analyzer with a Thinkjet* printer. Amplitude response, termed MAG by the analyzer, was done by inserting a pseudo-noise signal, which has flat frequency spectrum, and recording the spectrum of the output. The pseudo-noise was internal to the analyzer and hence synchronized to produce a smooth trace, but the filters having their own clocks were not synchronized, and irregular traces sometimes resulted. Vertical scale is indicated on the graphs, in most cases 6 dB/large division so direct comparison is possible. Horizontal scale is usually linear. Use of a linear scale makes high-pass filters appear sharper than low-pass, but is desirable for direct comparison to the phase plots. Filter frequency is usually 1000 rad/sec (about 160 Hz) and a full scale of 400 Hz used, showing slightly more than an octave above for the low-pass case.

Phase response was done similarly. Vertical scale is always 45 deg/large division, running from -180 degrees to +180 degrees. Horizontal scale is always the same as for the amplitude plot. The cursor is place at the filter frequency.

The step response, termed TIME by the analyzer, was done by converting the internally generated impulse to a step with a flip-flop. The time signal of the output was then recorded, normally with a delay of zero. Scales are of course linear. Vertical amplitude was normalized to four large divisions, starting from the bottom for low-pass but from the center for high-pass (and bandpass). Horizontal full scale is usually 16 msec. One cycle at the normalized frequency is about 6.3 msec, close to the cursor setting of 6.4 msec. Thus slightly more than two cycles are presented. The same settings were used as much as possible, but they don't make sense for some cases, and some examples were circuits already in use and hence hard-wired.

The circuits were constructed mostly using 1% capacitors and 2% resistors; sometimes tweaking was necessary to prevent oscillation or serious errors, mostly with the higher-order filters. FET-input op-amps (which are about an order of magnitude faster than 741's) were used. The curves at times will show noticeable

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deviations from the ideal. If exact curves are required the mathematical function must be plotted, which is done in some of the references, usually by computer. Some of the more pertinent aspects of some of the photographs will not be discussed.

The simple RC low-pass (Figure A-1) offers only about 6 dB rejection an octave above cutoff, a minimal filter. The phase shift appears linear only because it is so low; a straight edge will confirm this. The step response is a simple exponential, about 1/6 of the period due to the 2π factor. The 1-pole RC high-pass (Figure A-2) appears to have a sharper cutoff only because a linear frequency scale is used; any high-pass must go to zero ($-\infty$ dB) at zero frequency. Conversely, the high-pass cannot reach 0 dB on the picture as the low-pass does. If a log frequency scale were used, the low-pass and high-pass would be mirror images in most cases. The phase plot drops to zero at low frequency because there is insufficient amplitude to make a measurement. The step response is also a simple exponential, but falling, as a high-pass shows the initial step but must return to zero. The 4-pole equal-RC (Figure A-5) shows four times as much attenuation at an octave, but also at the cutoff. Note that at +180 degrees the phasemeter shifts to -180 degrees. Note also that for four poles phase response is identical for low-pass and high-pass. The step response shows no overshoot. The no-overshoot property does not carry over to the high-pass.

For the Butterworth curves (Figures A-7 through A-14), note that the attenuation at the cutoff frequency remains 3 dB but the attenuation at an octave improves with increasing order. On the other hand, ringing in the step response becomes worse.

The Chebyshyev curves (Figures A-15 through A-22) show how the cutoff slope may be made considerably steeper by allowing 1 dB ripple in the passband. However, the ringing in the step response is worse than Butterworth. Misadjustment is evident as the 1 dB limit is exceeded in some of the pictures. Filters with less than 1 dB ripple were not attempted. Three dB filters are not shown, as this is more ripple than is commonly used, and the improvement in steepness over 1 dB is not all that much.

The Bessel filters (Figures A-23 through A-26) show excellent phase linearity clear to the bandedge and beyond. The attenuation at an octave, though, is pitiful. The step response overshoot is barely detectable. The phase linearity does not carry over to the high-pass versions, so they are not shown.

The elliptics offer the steepest transition band. The 5-pole 4-zero low-pass (Figure A-27) hits the stopband about 20% past the cutoff frequency. The zeros are clearly evident. Note that the stopband ripple is not equal as it should be. A 5-pole 4-zero design having less ripple but more rejection, and hence less steepness, is shown in Figure A-28. Note that the scale is different and that the stopband is below the noise floor of the measuring system. The 5-pole 4-zero high-pass (Figure A-29) only appears steeper. The 7-pole 6-zero low-pass (Figure A-30) appears comparable only because of the distortion of the linear scale; it is really steeper than the 5-pole 4-zero high-pass. Phase shift and step response are similar to Chebyshev filters of the same steepness. (Compare Figures A-21 and A-30. The elliptic appears to have a faster time response primarily because the analyzer does not have a fast enough time sweep; the two cursors are set at equivalent points.) Figure A-31 shows the response of

a 7-pole 6 zero 1.25 dB/58 dB Elliptic built with switched-capacitor technology, so the cutoff frequency may be varied as shown. Note that the characteristic changes slightly as the frequency is varied; it should not. This type of filter is relatively noisy, so the stopband appears higher than it really is. In actual operation one may or may not get the full benefit of the true rejection level, depending on the system application. The actual circuit is shown in Figure A-32.

The constant-K filters (Figures A-33 and A-34) are reasonably well behaved. A good amplitude characteristic is obtained with no tweaking. The low-pass makes a good delay line; the phase response is nearly linear over most of the passband, and the step response exhibits a delay comparable to the rise time. The high pass is interesting in that the ringing is not a constant frequency, but slows down as it dies out. Figure A-35 shows the result of adding two zeros to a constant-K filter to reject two specific frequencies. This is easily done because the passive prototypes are similar. The result is similar to an elliptic, but the ripple is unequal since it was the frequency of the notches that was specified. The notches cause a slight droop at the edge of the passband, but this may be compensated to a certain extent by modifying the terminations (see Reference A-1). The phase linearity is still quite good. Note the altered scales. The actual circuit is shown in Figure A-36.

The good phase linearity of the Lerner filters (Figures A-37 through A-39) is evident. That of the high-pass is less than ideal, but it is the best shown yet for a high pass. Step response is meaningful only for the low-pass. Again we have a good delay line; observe the "pre-shoot." The sweeps here have been lengthened to show the details of the wiggles. Cutoff frequency is not normalized.

Figure A-40 shows the response of an Elliptic bandpass using floating synthetic inductors, after considerable tweaking. Ripple misses the specification significantly. Log frequency scale is used to show the symmetry of the transformation. Figure A-41 is the response with the components rearranged into a band-reject. Linear frequency scale is used here. Figure A-42 is for a bandpass made simply by combining high-pass and low-pass sections. Performance cannot be as good as that of Figure A-40, since the latter is optimum.

The narrowband response (Figure A-43) was obtained from the equal-capacitor version. The phase response rises sharply by 180 degrees as the peak is traversed. The response of the narrowband filter to any input is, of course, a sine wave at the center frequency. The height of each peak can be seen to be diminishing. The initial amplitude is not normalized, as it depends on Q. The notch filter (Figure A-44) was constructed using the Wein-bridge circuit. The depth of the notch depends on the accuracy of the components. The phase shift jumps 180 degrees going through the notch. It might be surprising that the step response rings at the center frequency, since that is the frequency at which the filter transmits least. One way to explain this is to observe that the ringing is upside down; what we are seeing is all frequencies (the step) except (minus) the center frequency. Log frequency scale is used in both cases; the plots look strange otherwise.

Figure A-47 shows the response of an eight-section commutating filter having a center frequency of 160 Hz and a bandwidth of 0.4 Hz, equivalent to a Q of 400. Note that the filter bandwidth is less than the analyzer bandwidth, so the

amplitude plot is not accurate. The phase plot shows an almost vertical jump at the center frequency(s) because the filter is so narrow. The step response reflects the additional passband at DC; it is an exponential corresponding to the RC low-pass, but "ratchets" up in steps corresponding to the repetition (center) frequency. The circuit is shown in Figure A-48.

Figure A-49 gives the response of a demonstration eight-stage transversal filter using a digital shift register instead of analog, which effectively clips the input. Taps were adjusted to give one cycle of a sine wave. The frequency response is basically a sin f/f centered at the repetition frequency, complicated by the addition of the mirror image at negative frequency. The phase response plot is ragged due to the interaction of the sampling frequencies of the filter and the analyzer, but the linear phase property is evident. The impulse is of interest here rather than step, and can be shown because it is really a pulse response, which is physically possible (as opposed to impulse). The actual circuit is shown in Figure A-50.

The amplitude response of the all-pass (Figure A-45) is decidedly uninteresting. The phase is the arctangent curve, exactly double that of the simple RC low-pass. The step response is interesting in that it initially steps in the wrong direction, due to the inversion at high frequencies, then recovers exponentially to the final value. Vertical scale is shifted and horizontal is delayed to give a good picture. Figure A-46 shows the performance of the phase-difference network, two 2-pole 2-zero networks whose outputs are 90 degrees apart across the frequency range 200 Hz-1400 Hz. Phase shift is shown for each output and the difference between them which is nearly constant 90 degrees. Log frequency scale is used, as the construction is basically logarithmic. The falling portions of the curves should be vertical; the finite slope results from interpolation by the analyzer being exaggerated due to the conversion to log scale.

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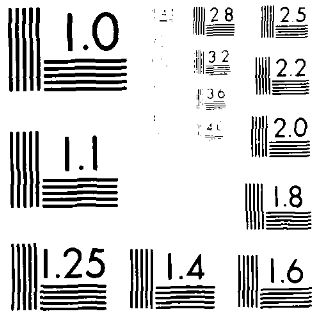


MICROCOPY RESOLUTION TEST CHART

AMPLITUDE

PHASE

STEP

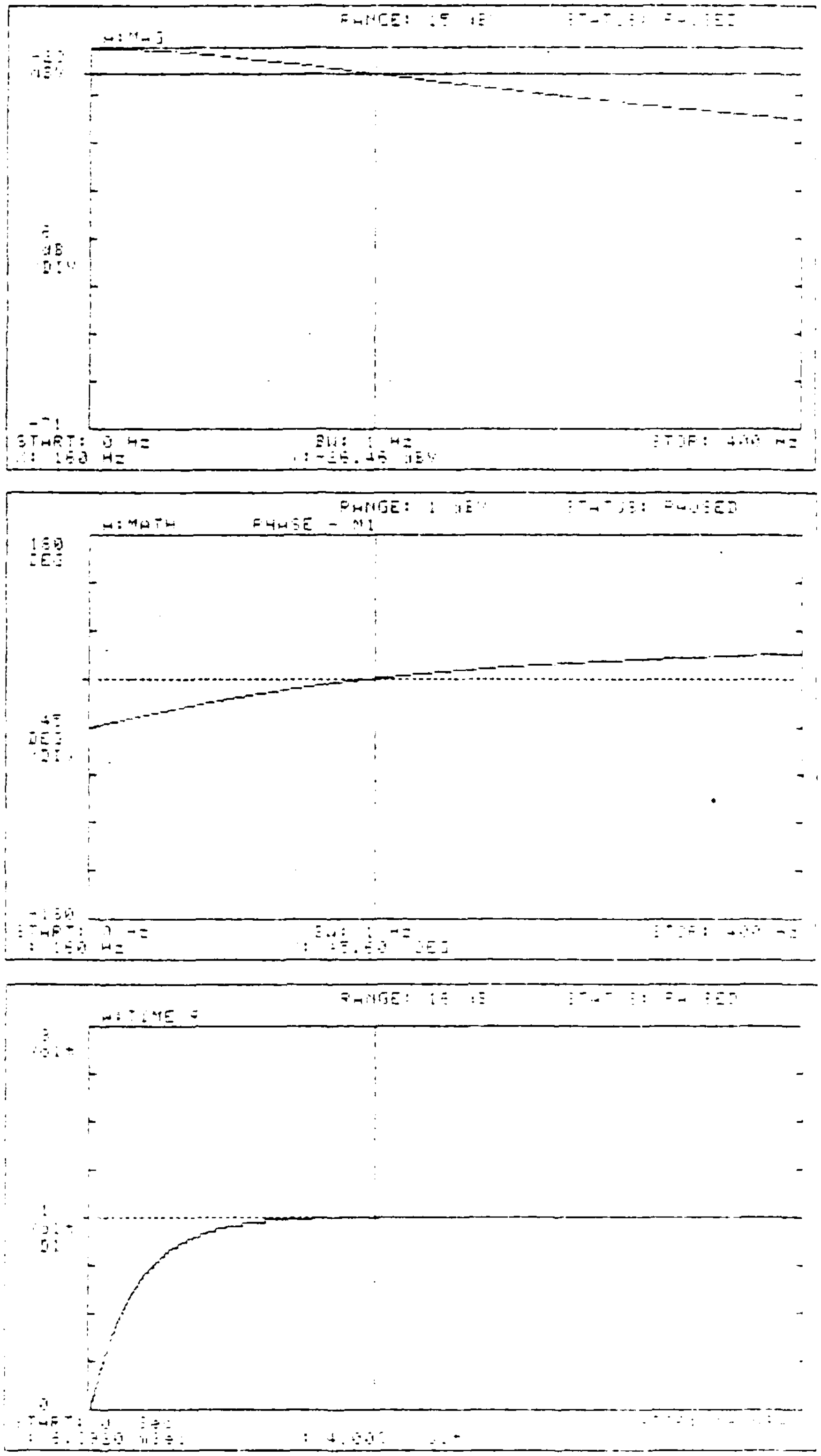


FIGURE A-1. 1-POLE RC LOW-PASS RESPONSE

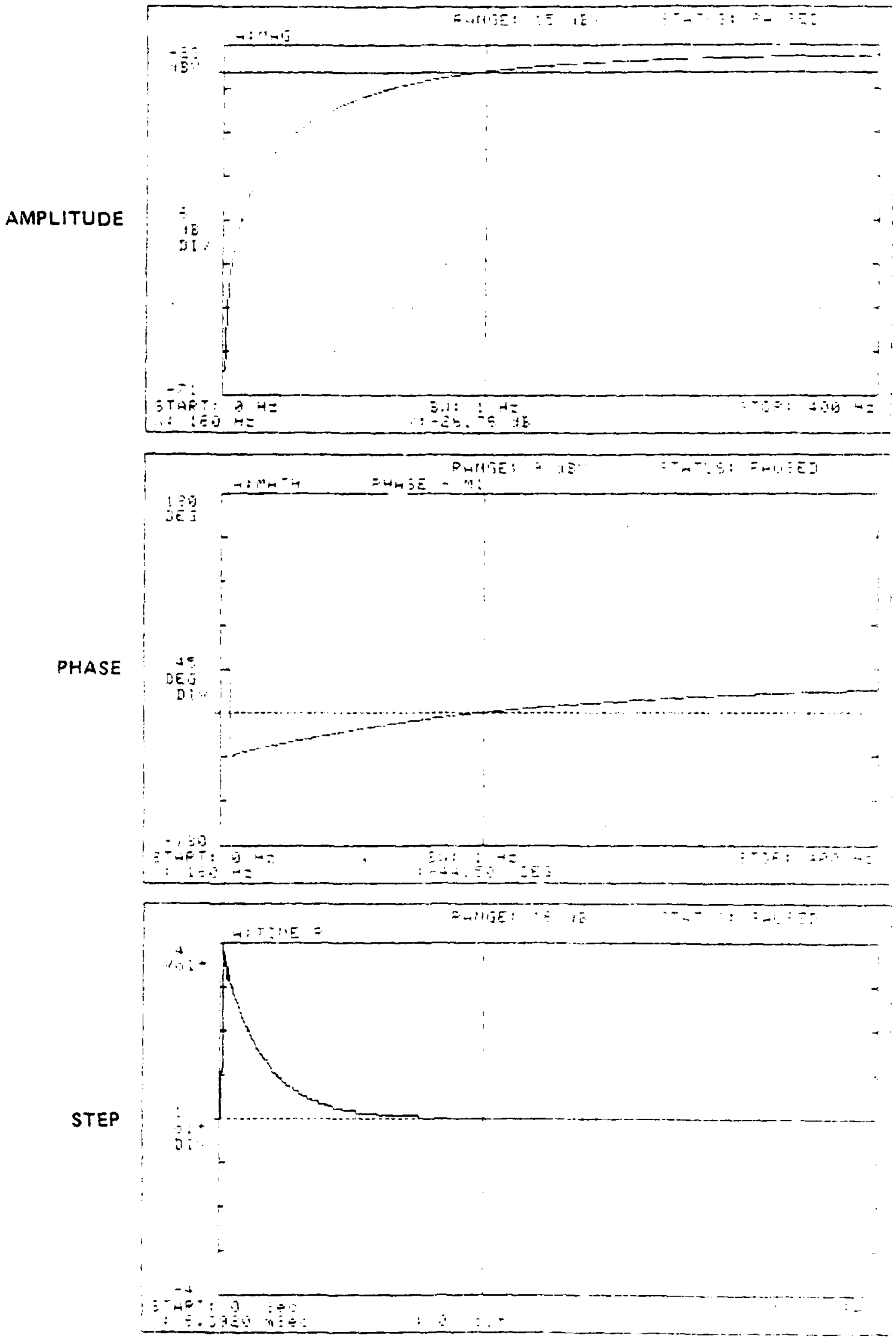


FIGURE A-2. 1-POLE RC HIGH-PASS RESPONSE

AMPLITUDE

PHASE

STEP

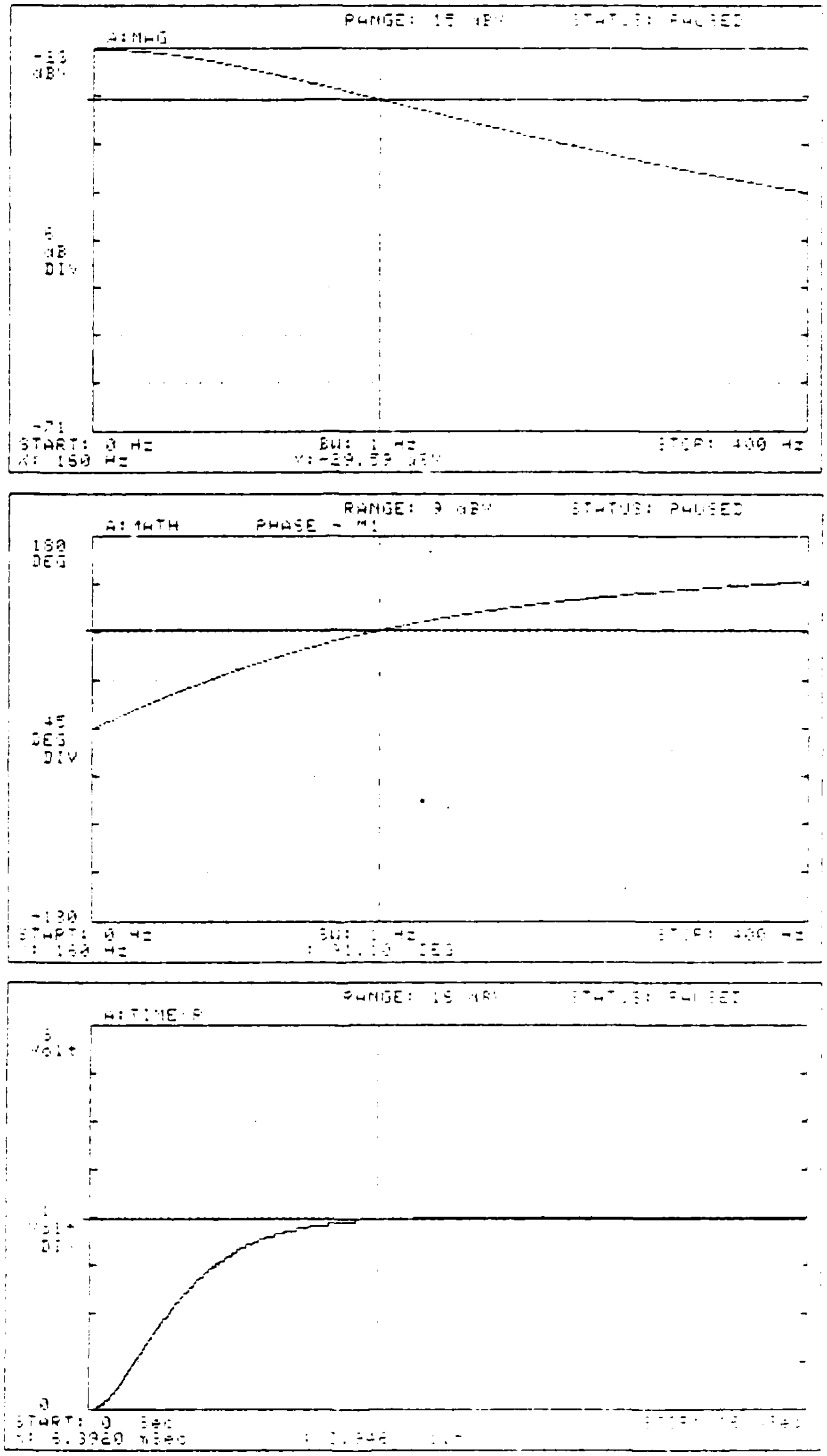


FIGURE A-3. 2-POLE EQUAL-RC LOW-PASS RESPONSE

AMPLITUDE

PHASE

STEP

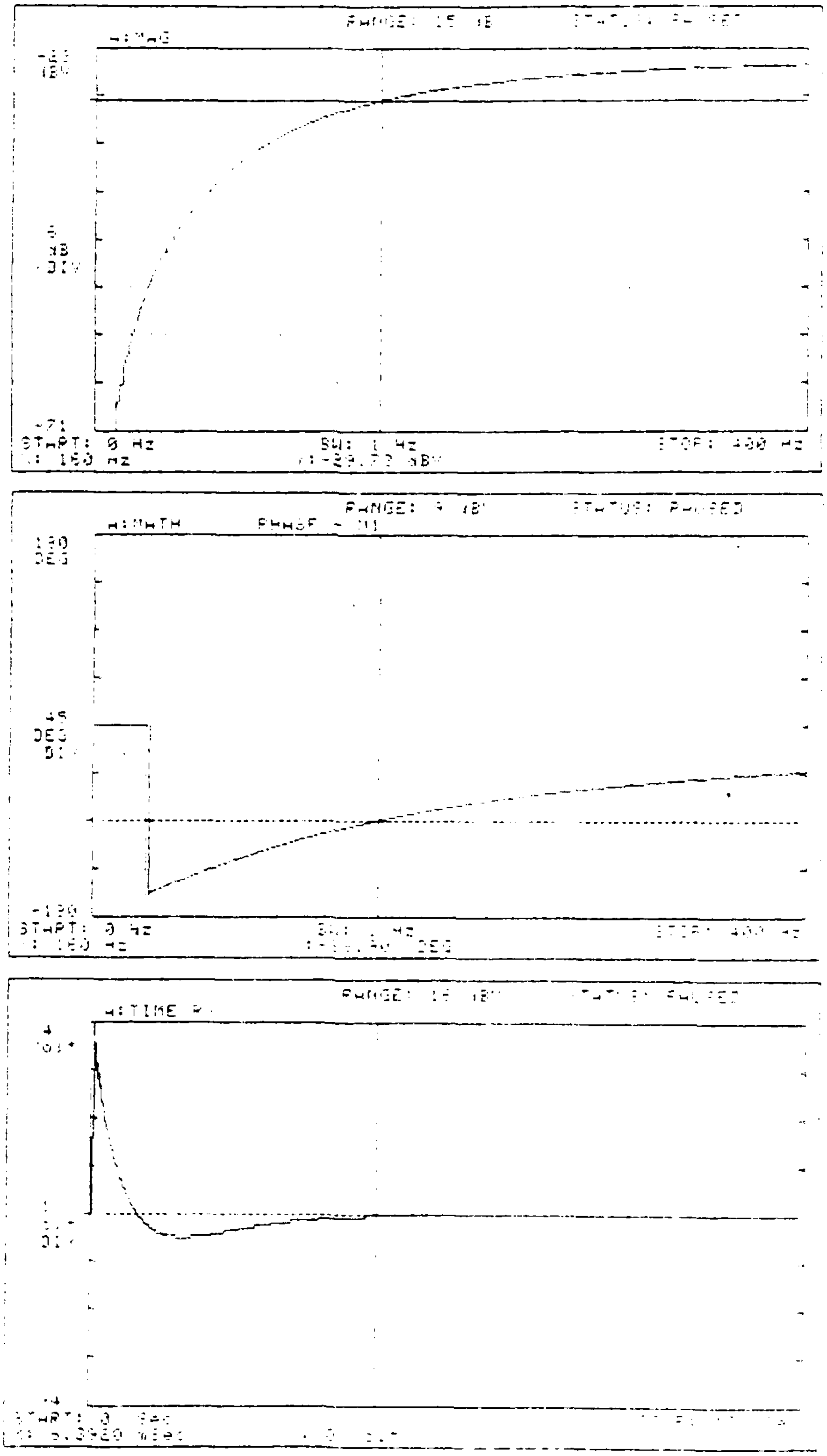


FIGURE A-4. 2-POLE EQUAL-RC HIGH-PASS RESPONSE

AMPLITUDE

PHASE

STEP

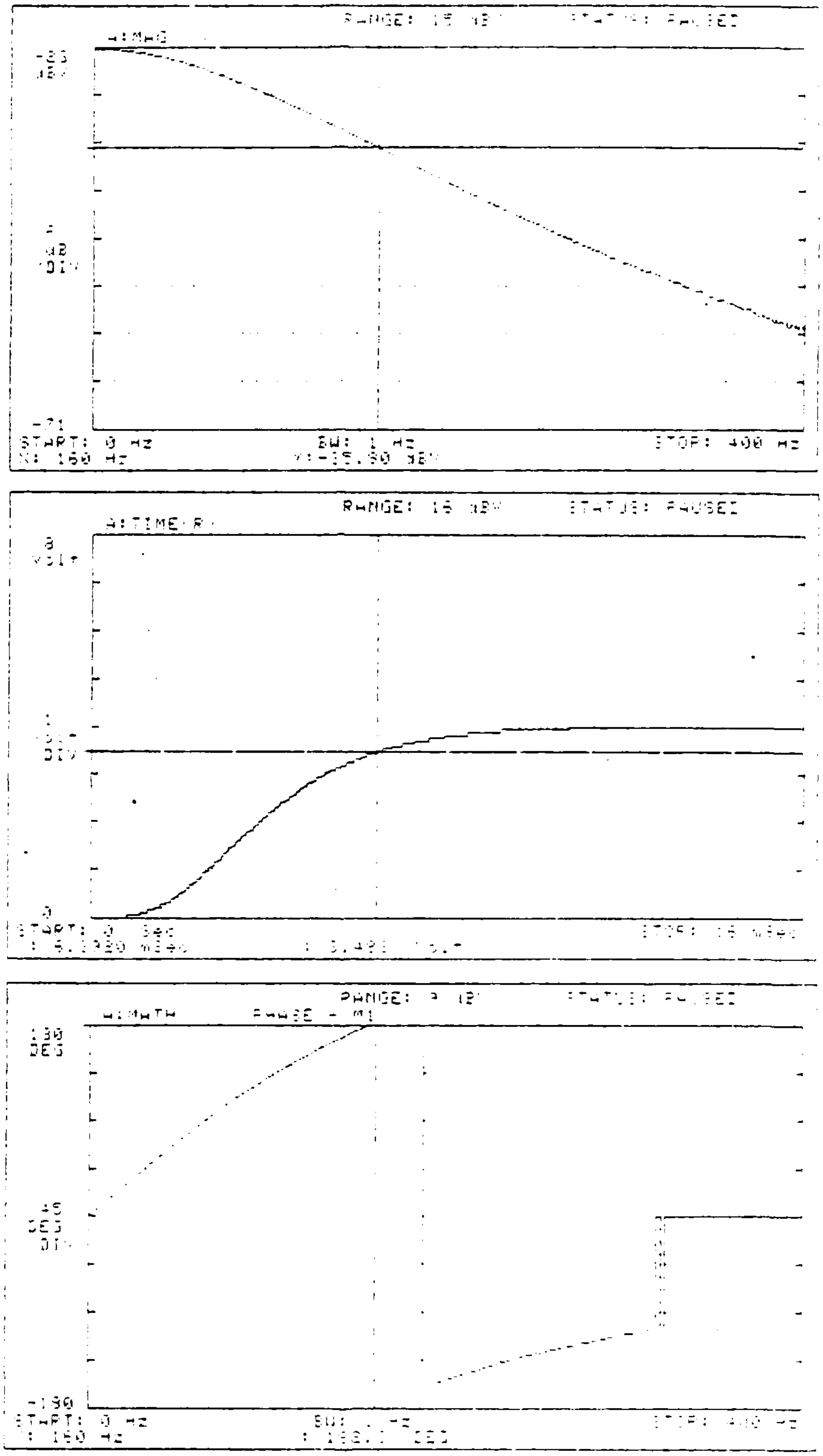


FIGURE A-5. 4-POLE EQUAL-RC LOW-PASS RESPONSE

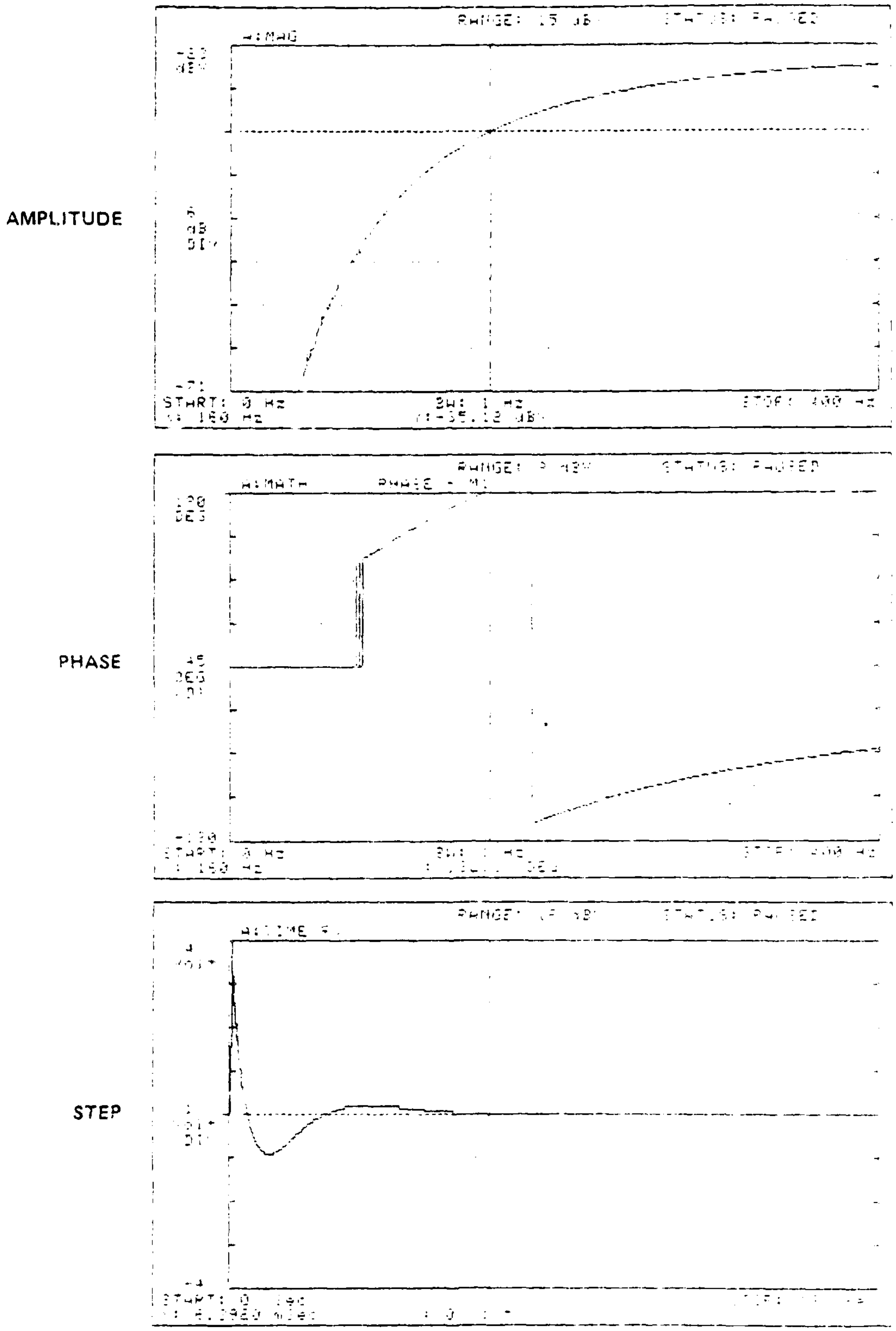


FIGURE A-6. 4-POLE EQUAL-RC HIGH-PASS RESPONSE

AMPLITUDE

PHASE

STEP

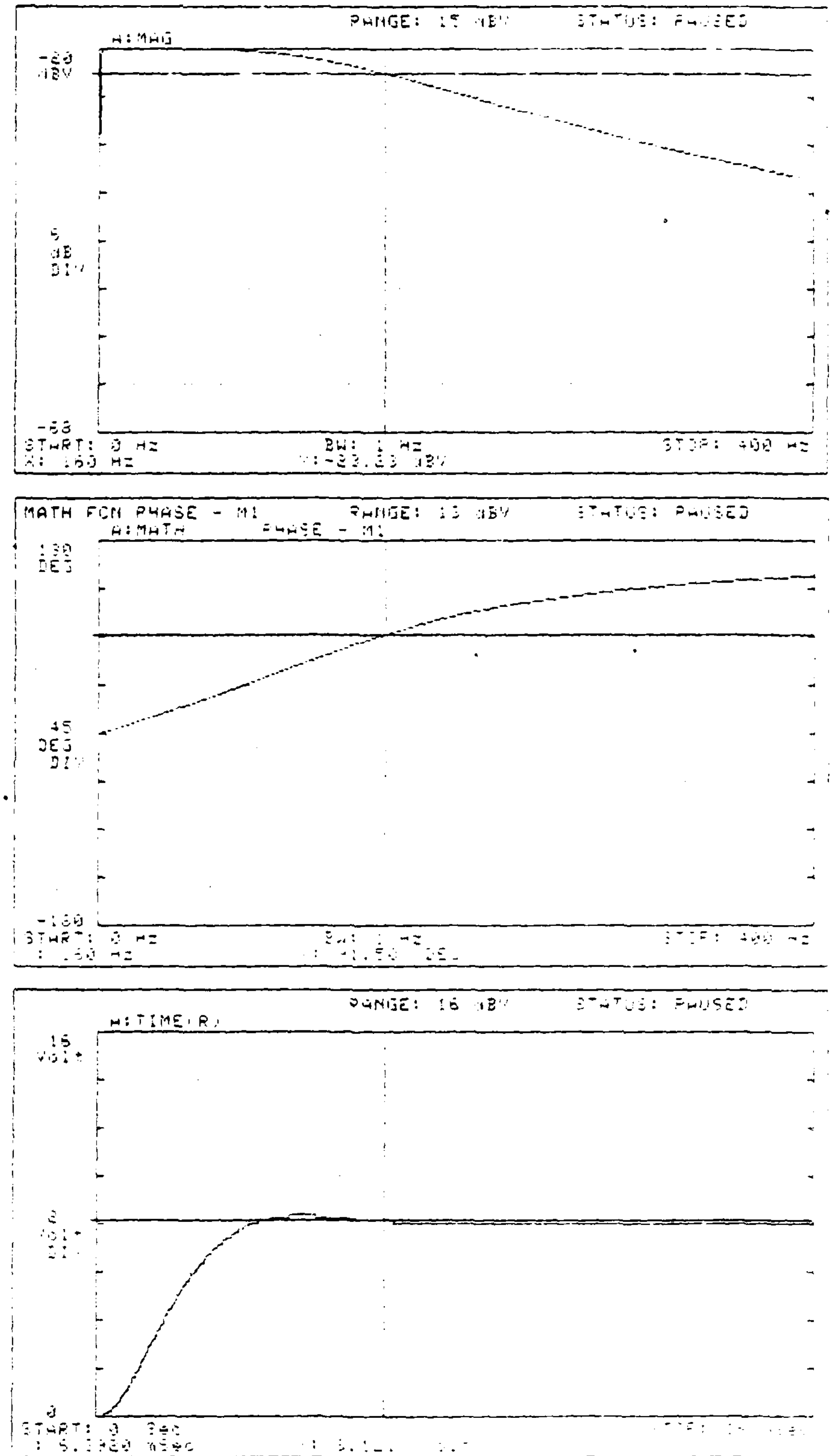


FIGURE A-7. 2-POLE BUTTERWORTH LOW-PASS RESPONSE

AMPLITUDE

PHASE

STEP

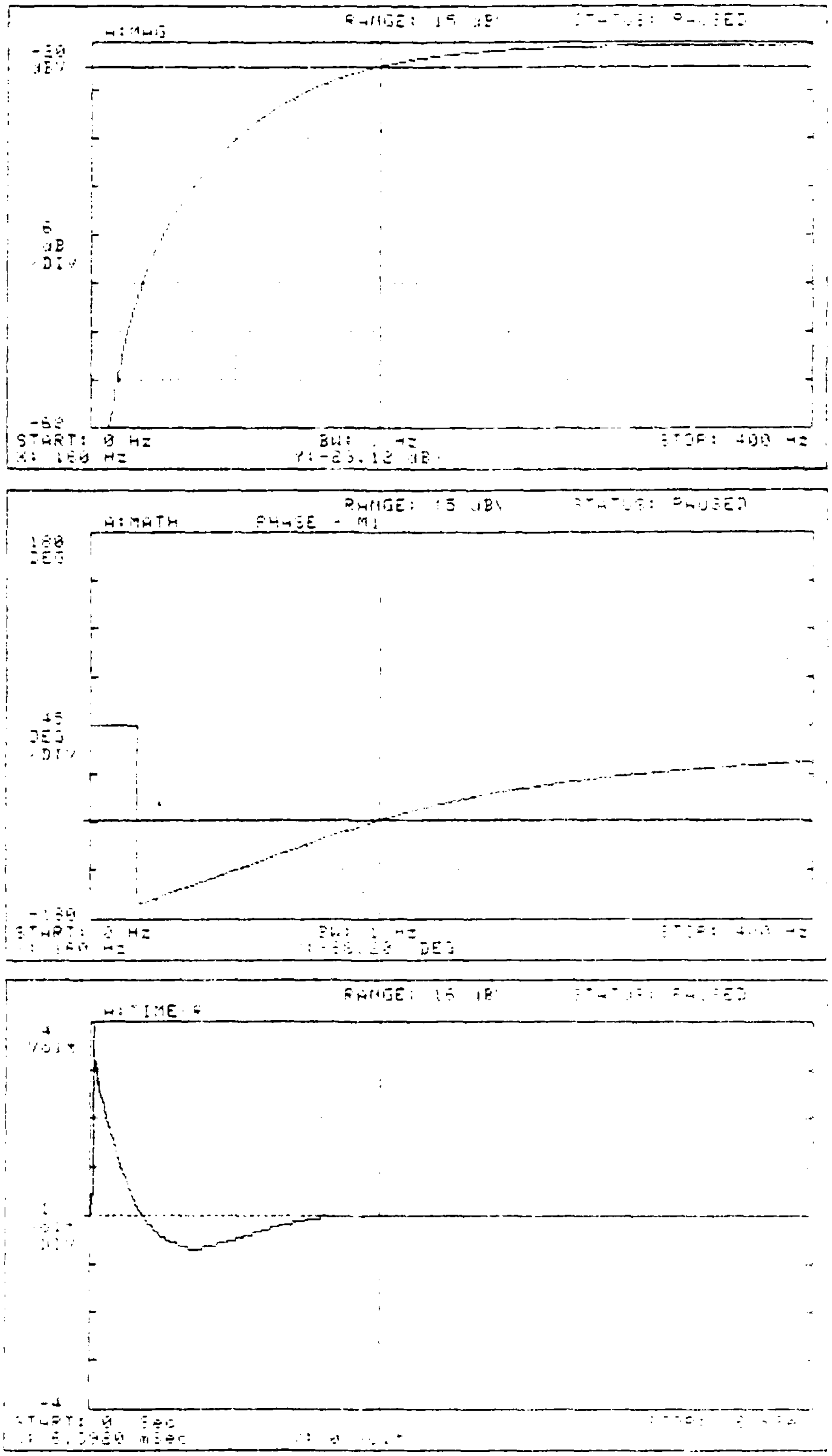


FIGURE A-8. 2-POLE BUTTERWORTH HIGH-PASS RESPONSE

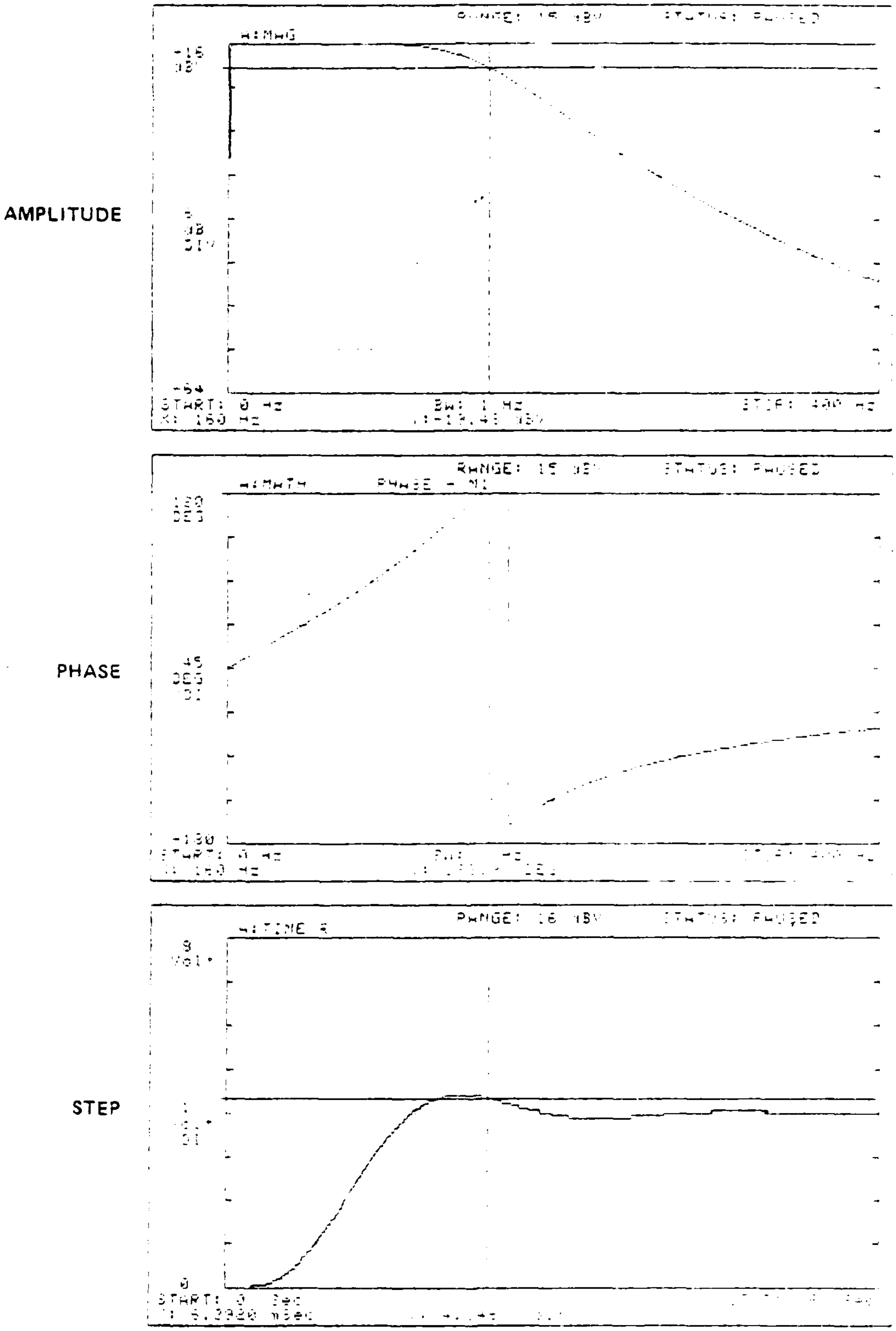


FIGURE A-9. 4-POLE BUTTERWORTH LOW-PASS RESPONSE

AMPLITUDE

PHASE

STEP

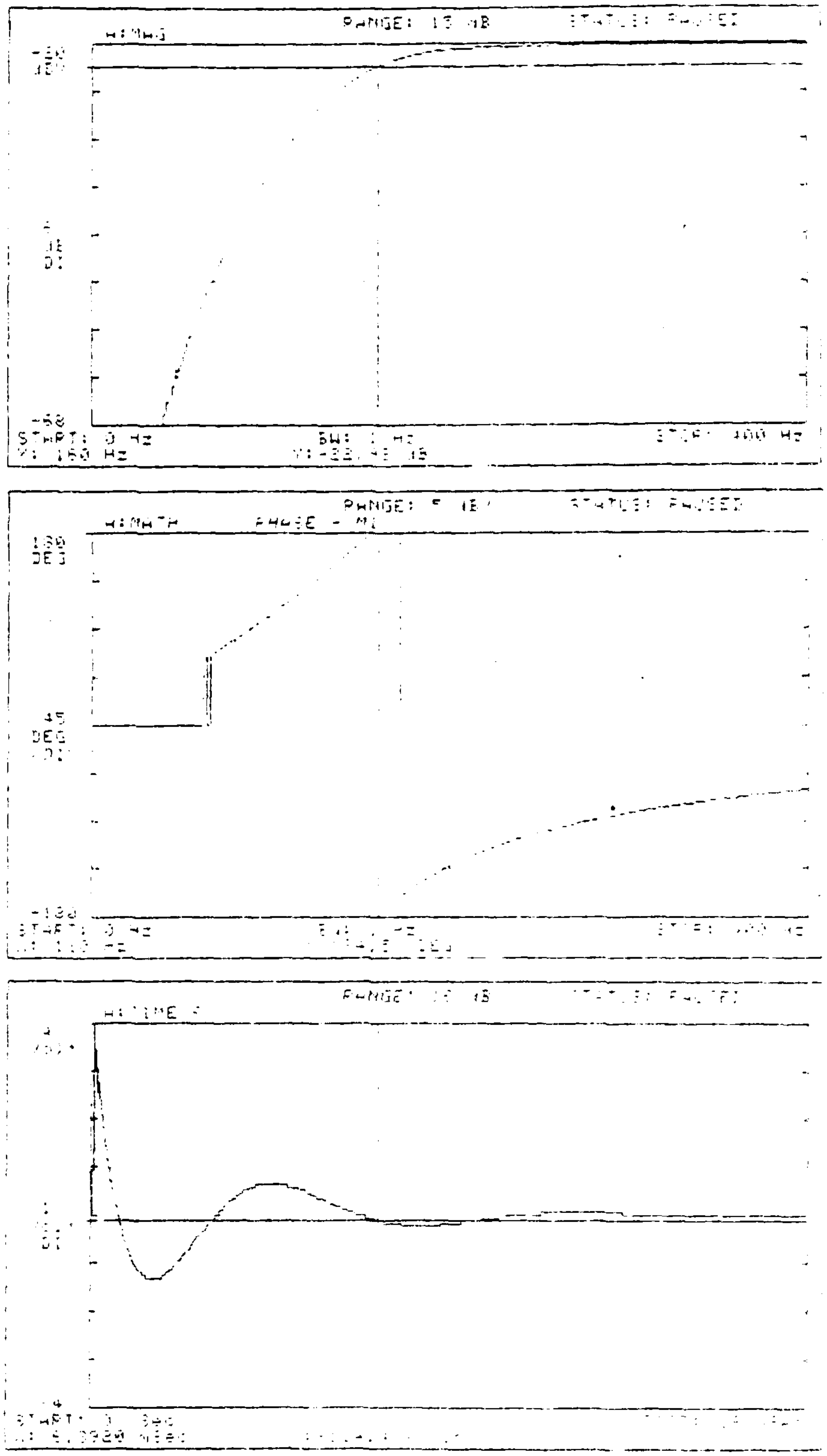


FIGURE A-10. 4-POLE BUTTERWORTH HIGH-PASS RESPONSE

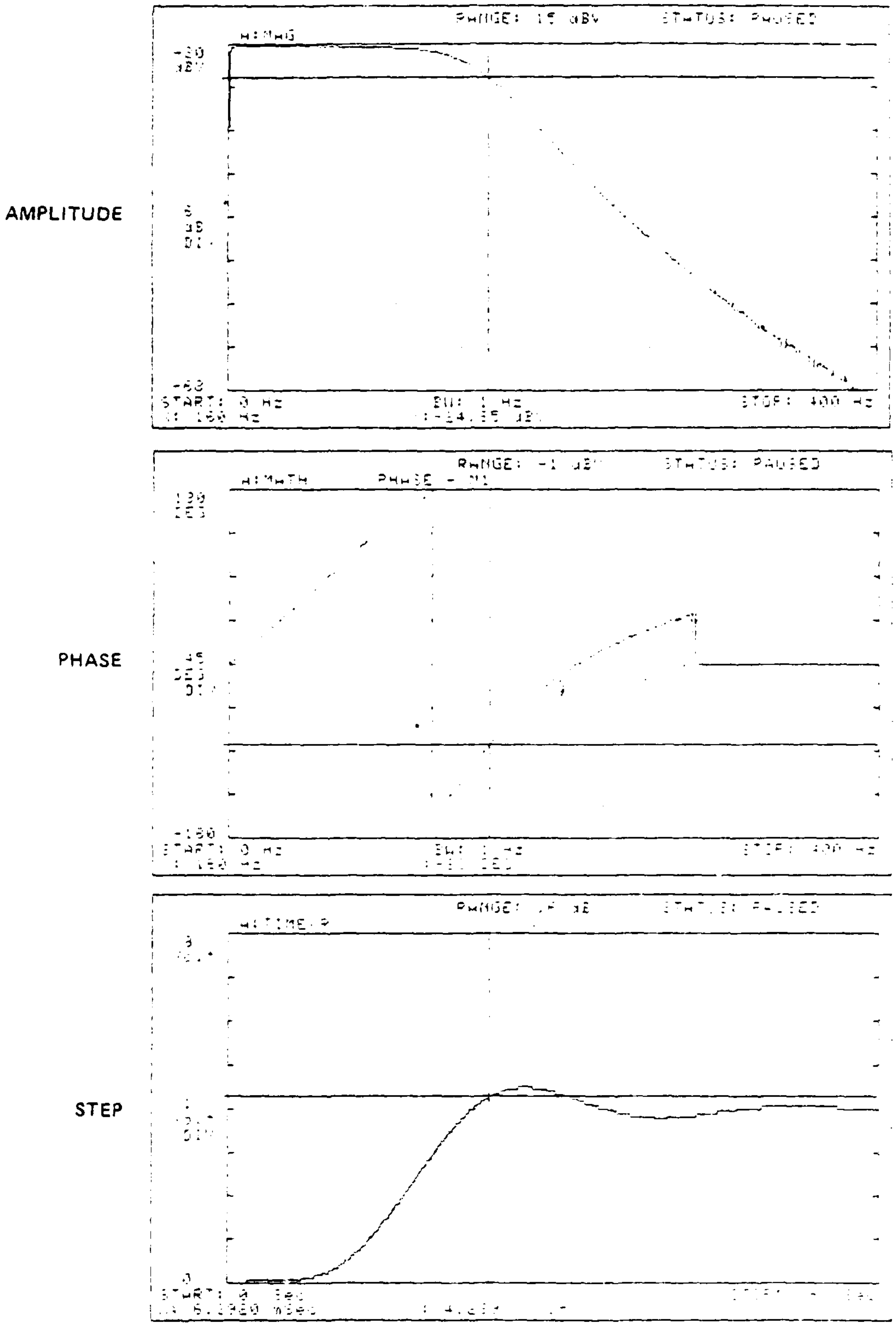


FIGURE A-11. 6-POLE BUTTERWORTH LOW-PASS RESPONSE

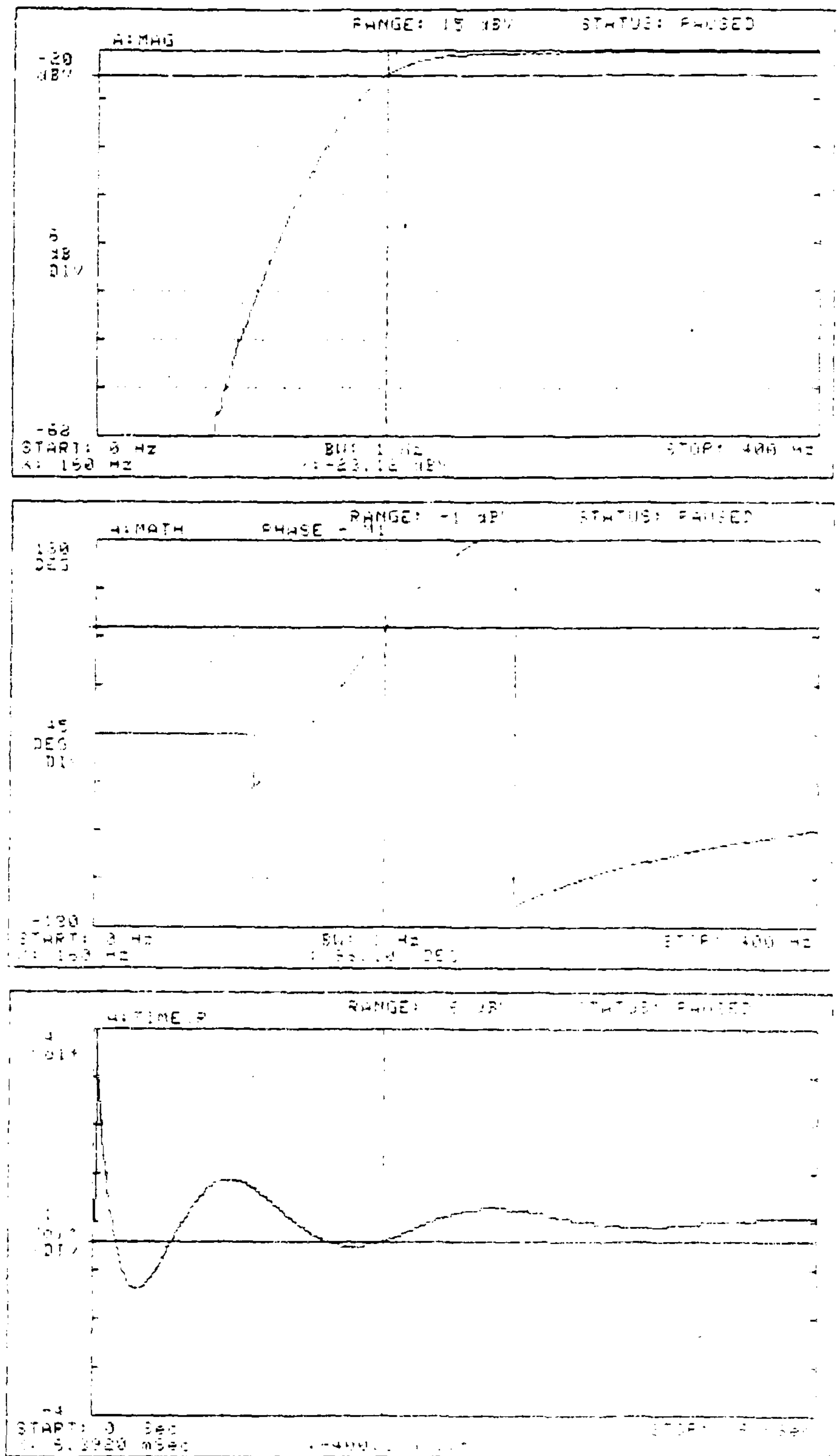


FIGURE A-12. 6-POLE BUTTERWORTH HIGH-PASS RESPONSE

AMPLITUDE

PHASE

STEP

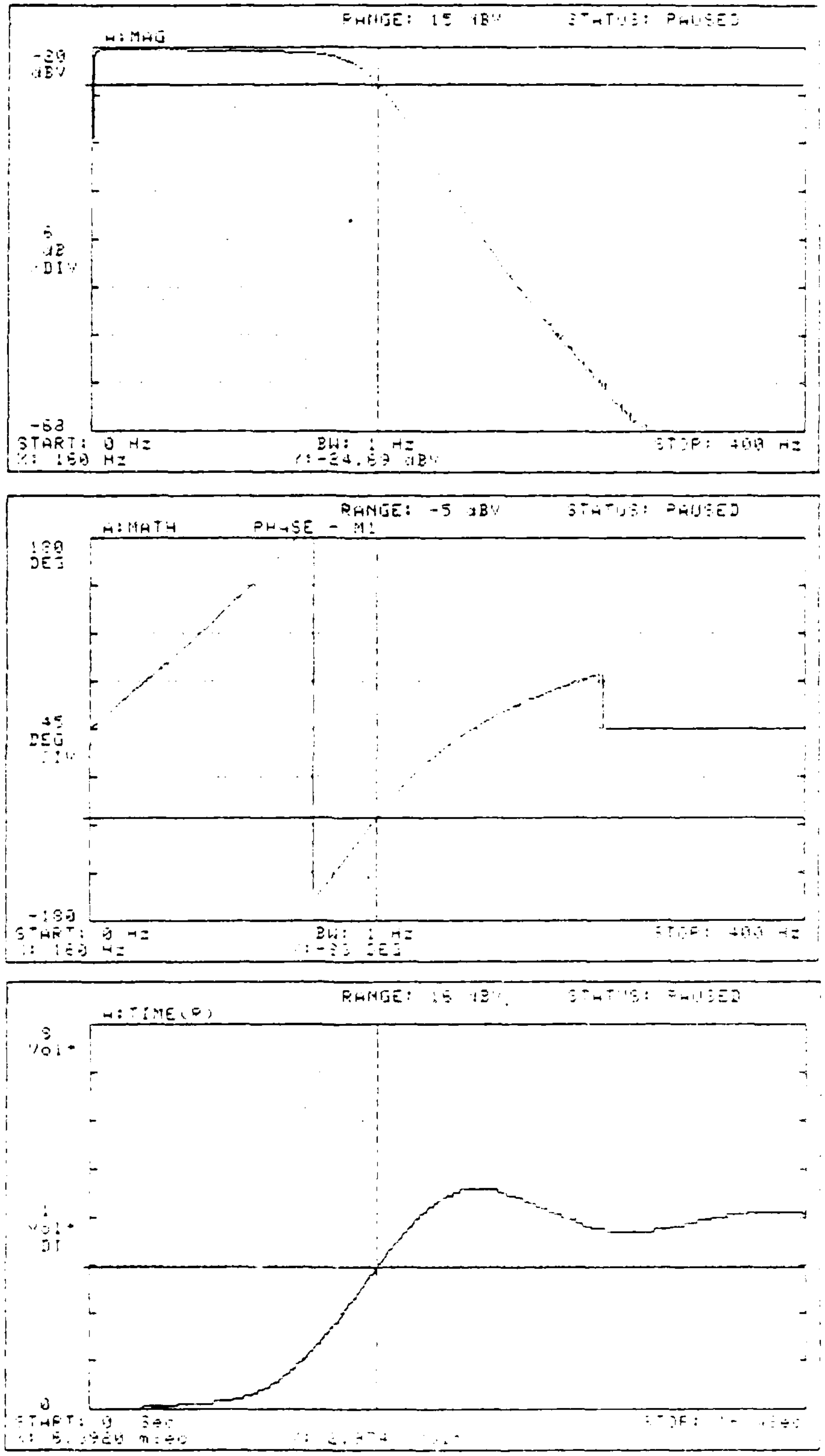


FIGURE A-13. 8-POLE BUTTERWORTH LOW-PASS RESPONSE

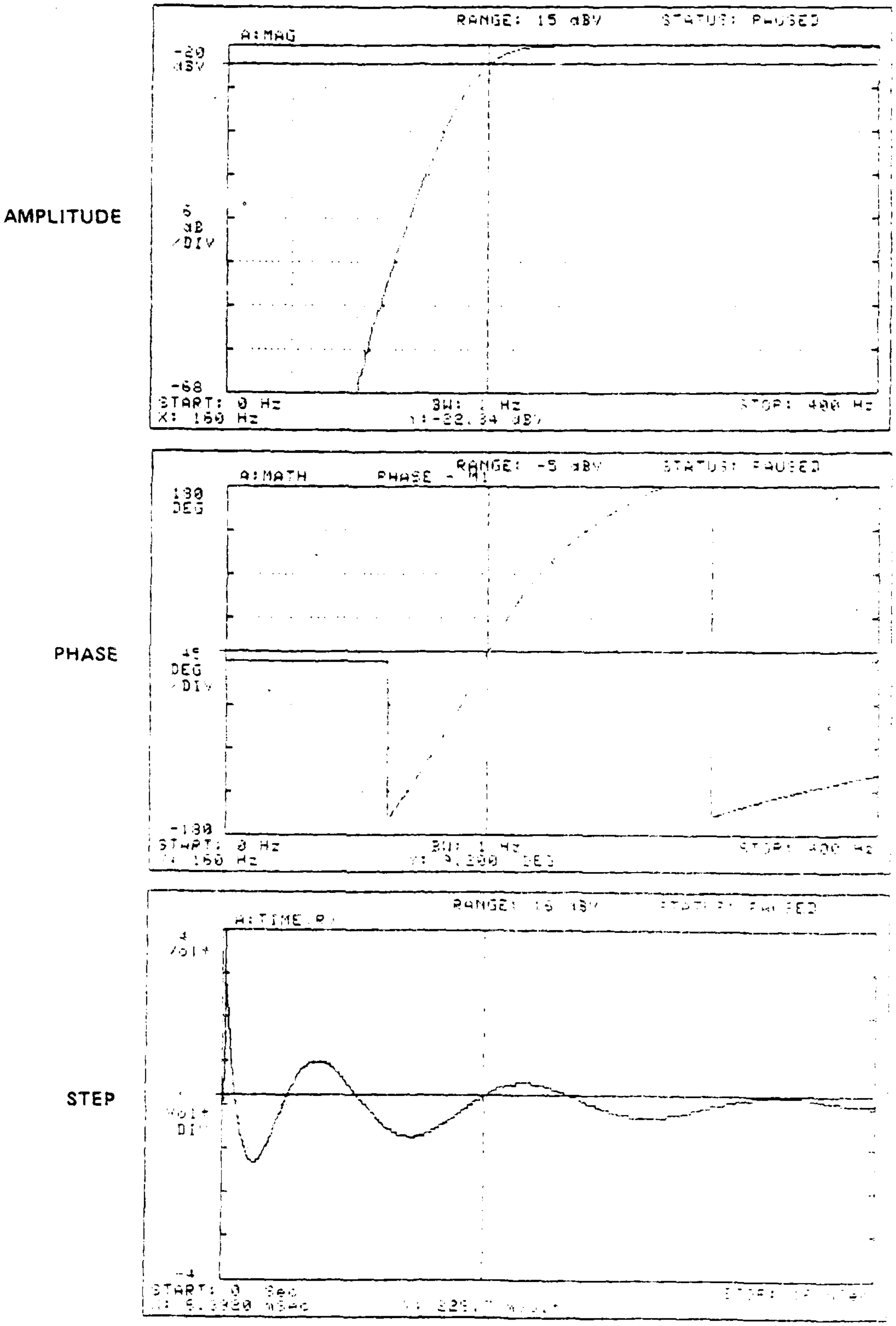


FIGURE A-14. 8-POLE BUTTERWORTH HIGH-PASS RESPONSE

AMPLITUDE

PHASE

STEP

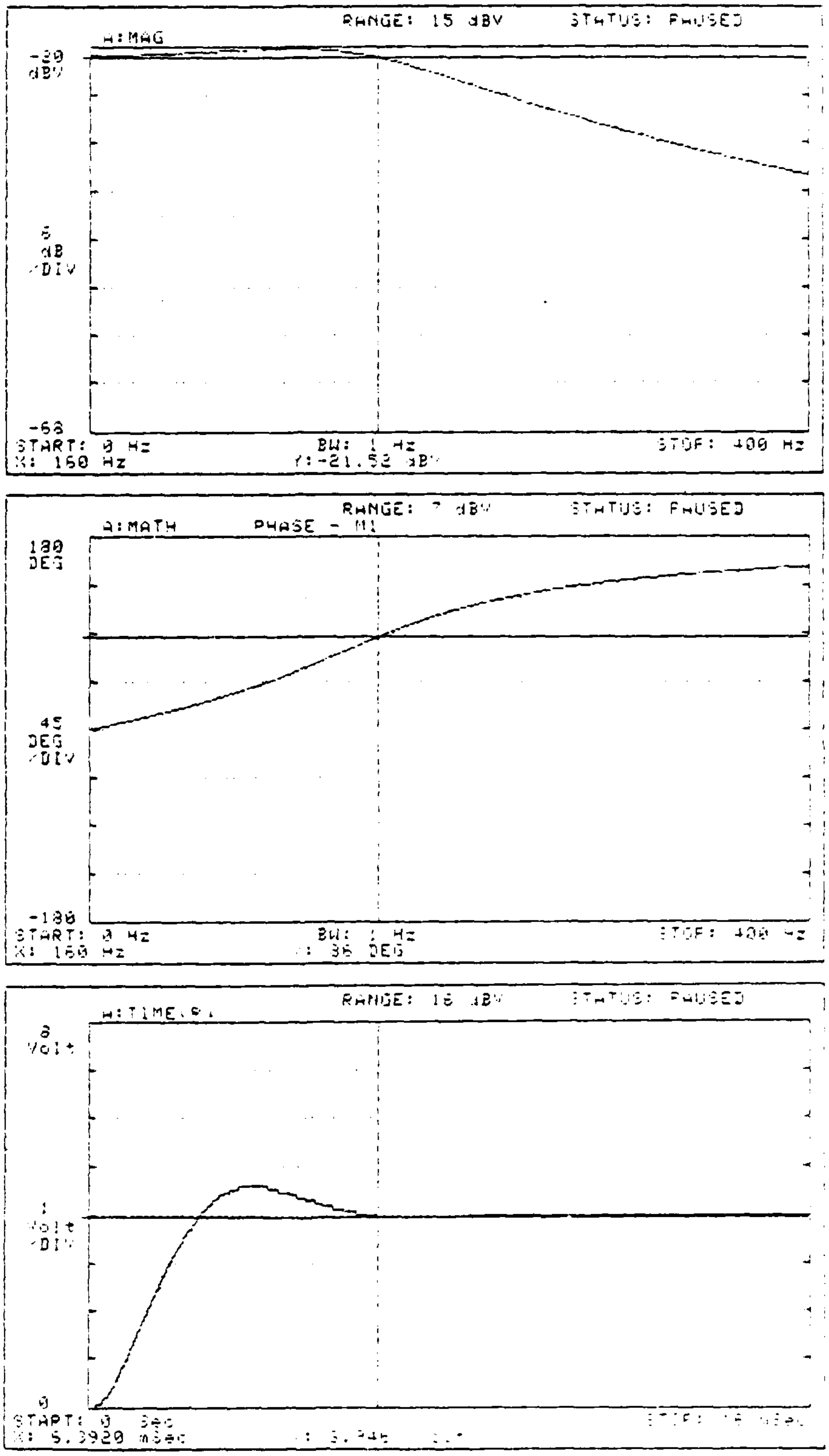


FIGURE A-15. 2-POLE 1dB RIPPLE CHEBYSHEV LOW-PASS RESPONSE

AMPLITUDE

PHASE

STEP

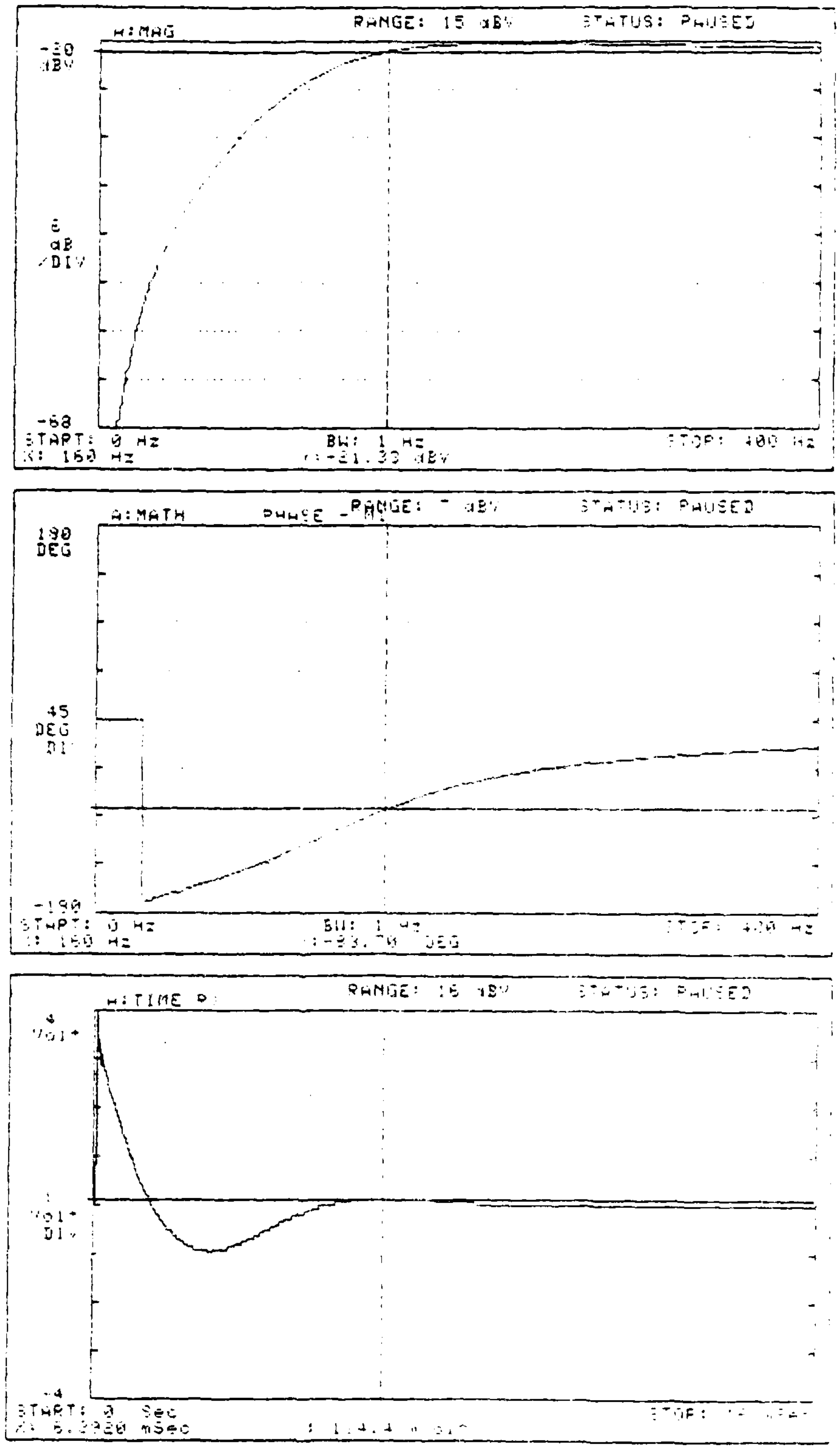


FIGURE A-16. 2-POLE 1dB RIPPLE CHEBYSHEV HIGH-PASS RESPONSE

AMPLITUDE

PHASE

STEP

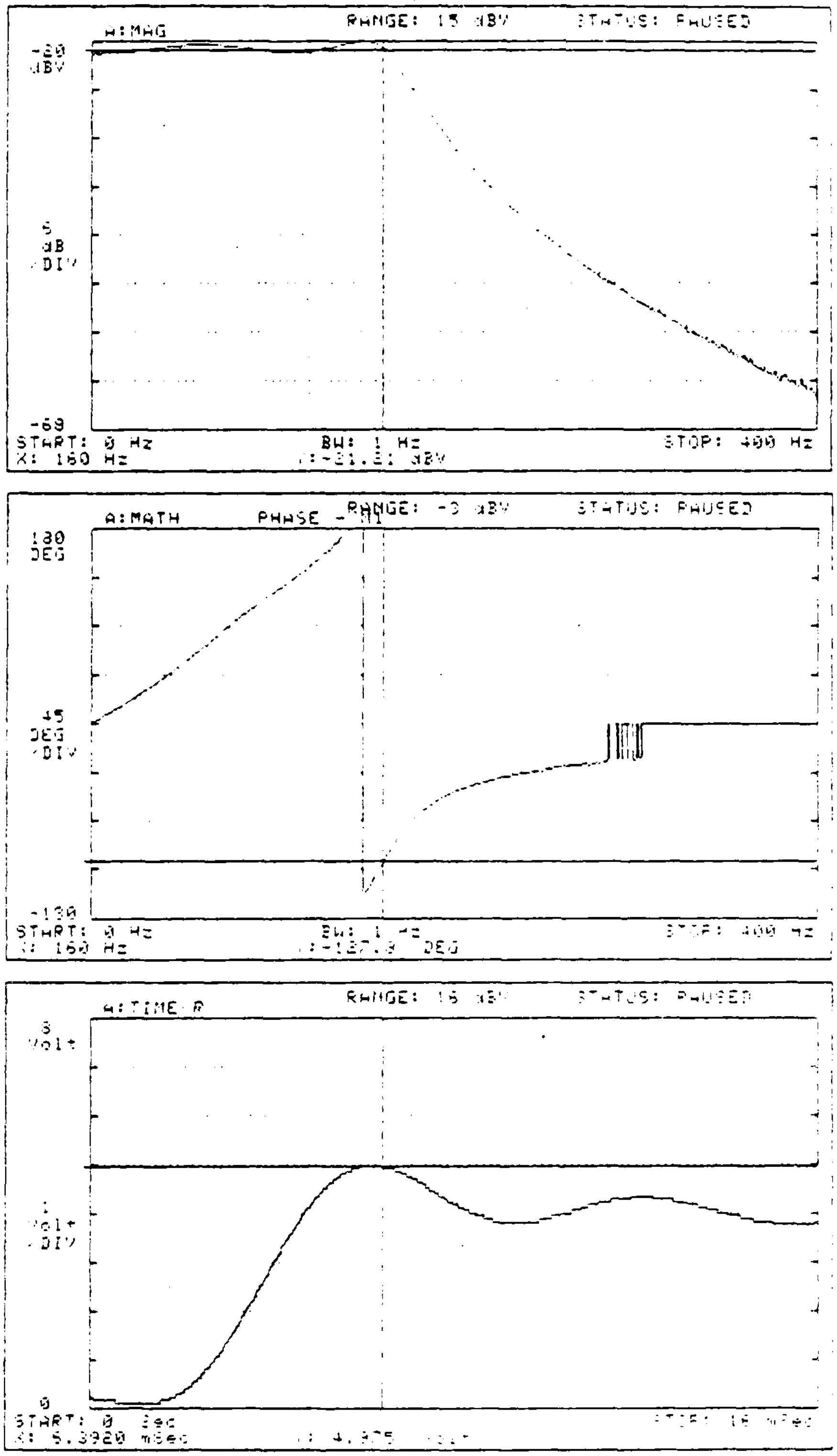


FIGURE A-17. 4-POLE 1dB RIPPLE CHEBYSHEV LOW-PASS RESPONSE

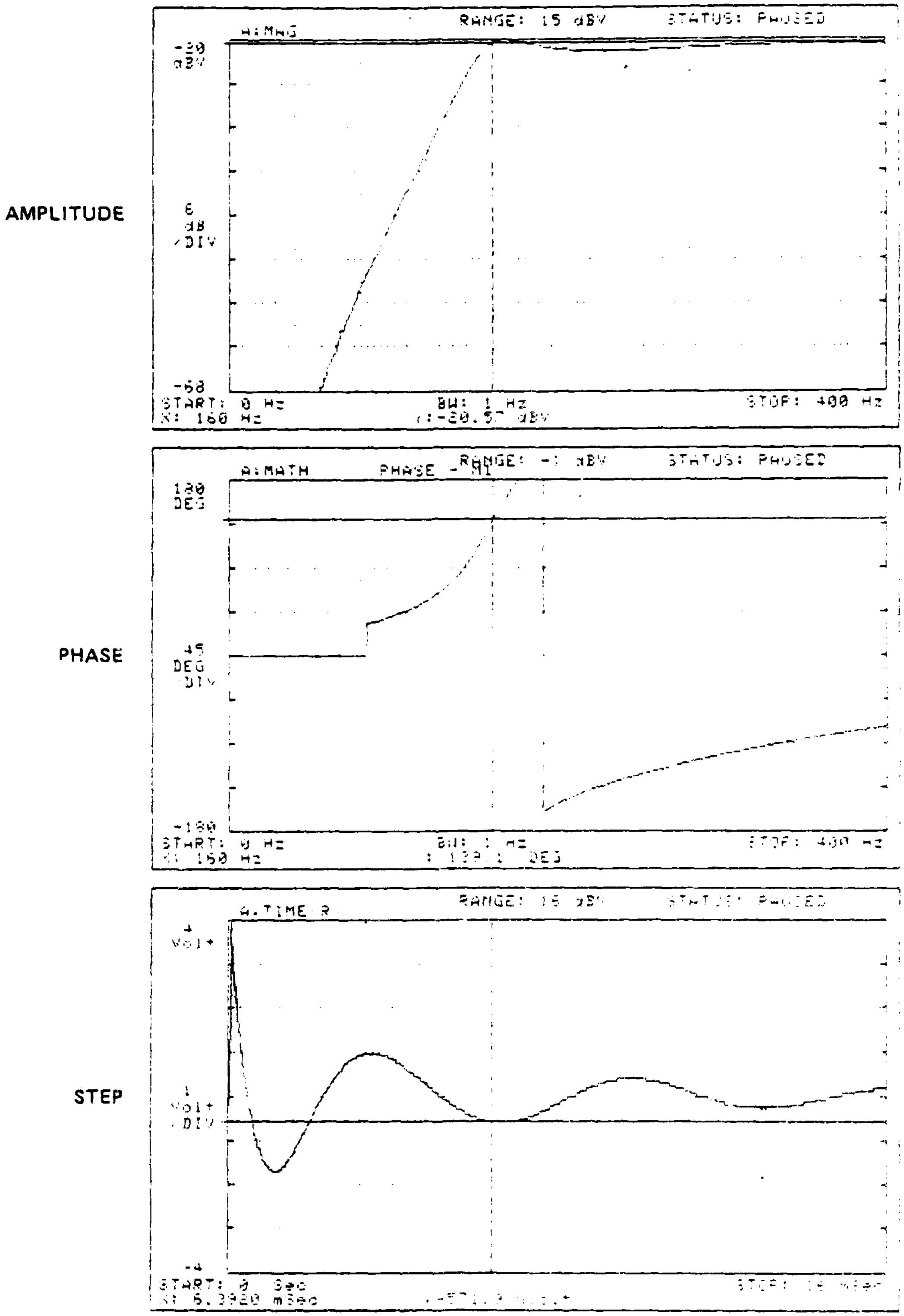


FIGURE A-18. 4-POLE 1dB RIPPLE CHEBYSHEV HIGH-PASS RESPONSE

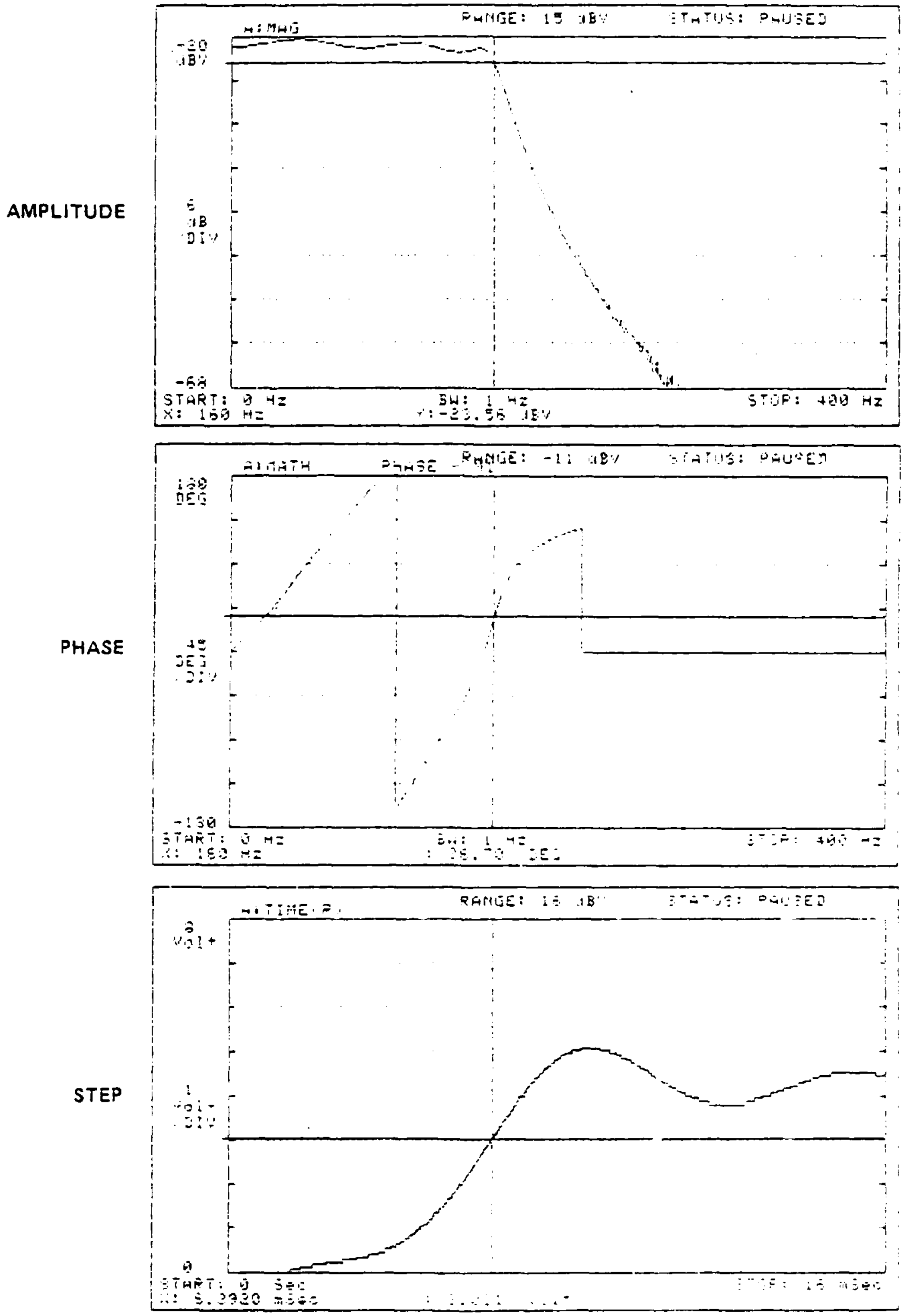


FIGURE A-19. 6-POLE 1dB RIPPLE CHEBYSHEV LOW-PASS RESPONSE

AMPLITUDE

PHASE

STEP

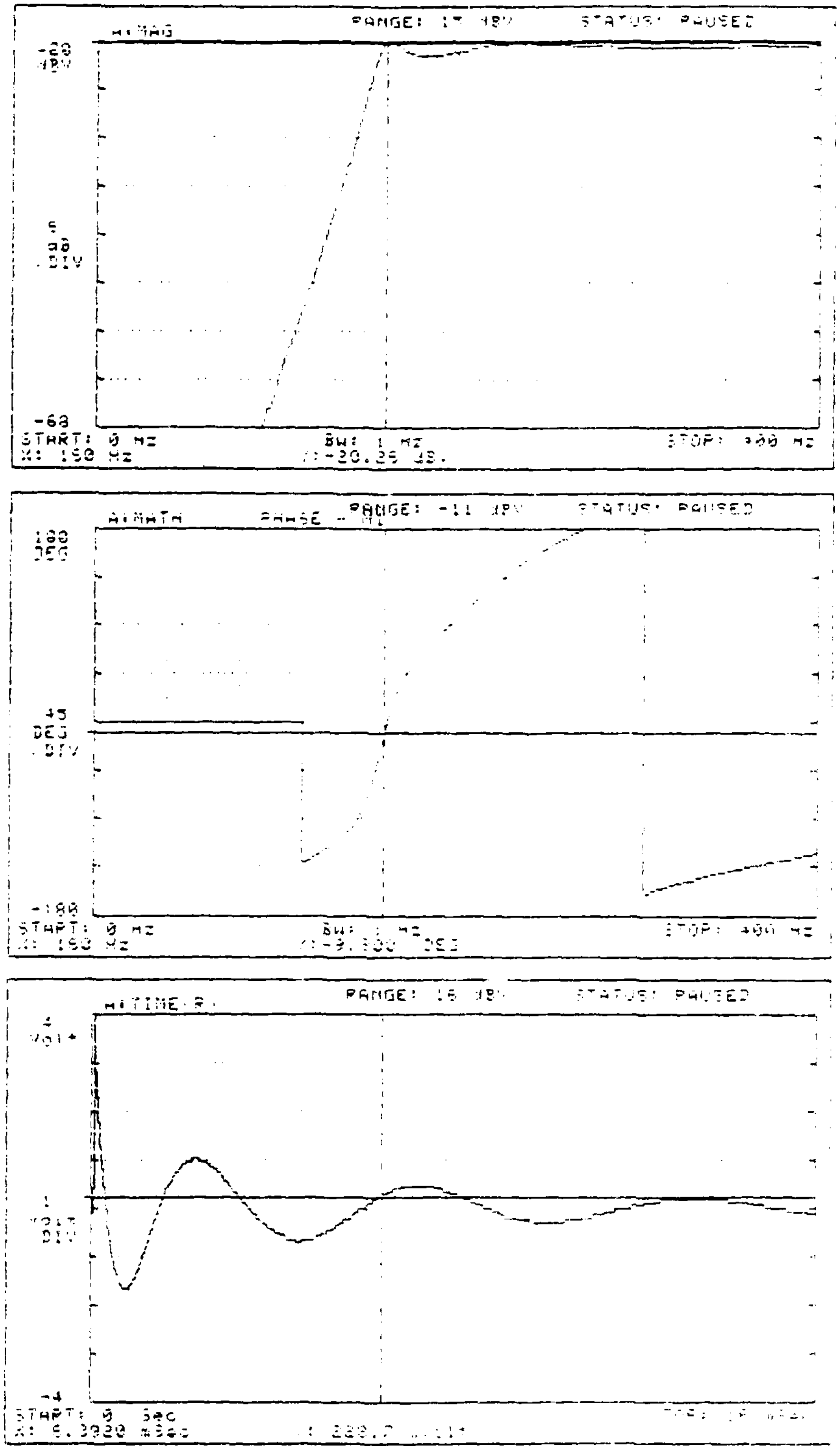


FIGURE A-20. 6-POLE 1dB RIPPLE CHEBYSHEV HIGH-PASS RESPONSE

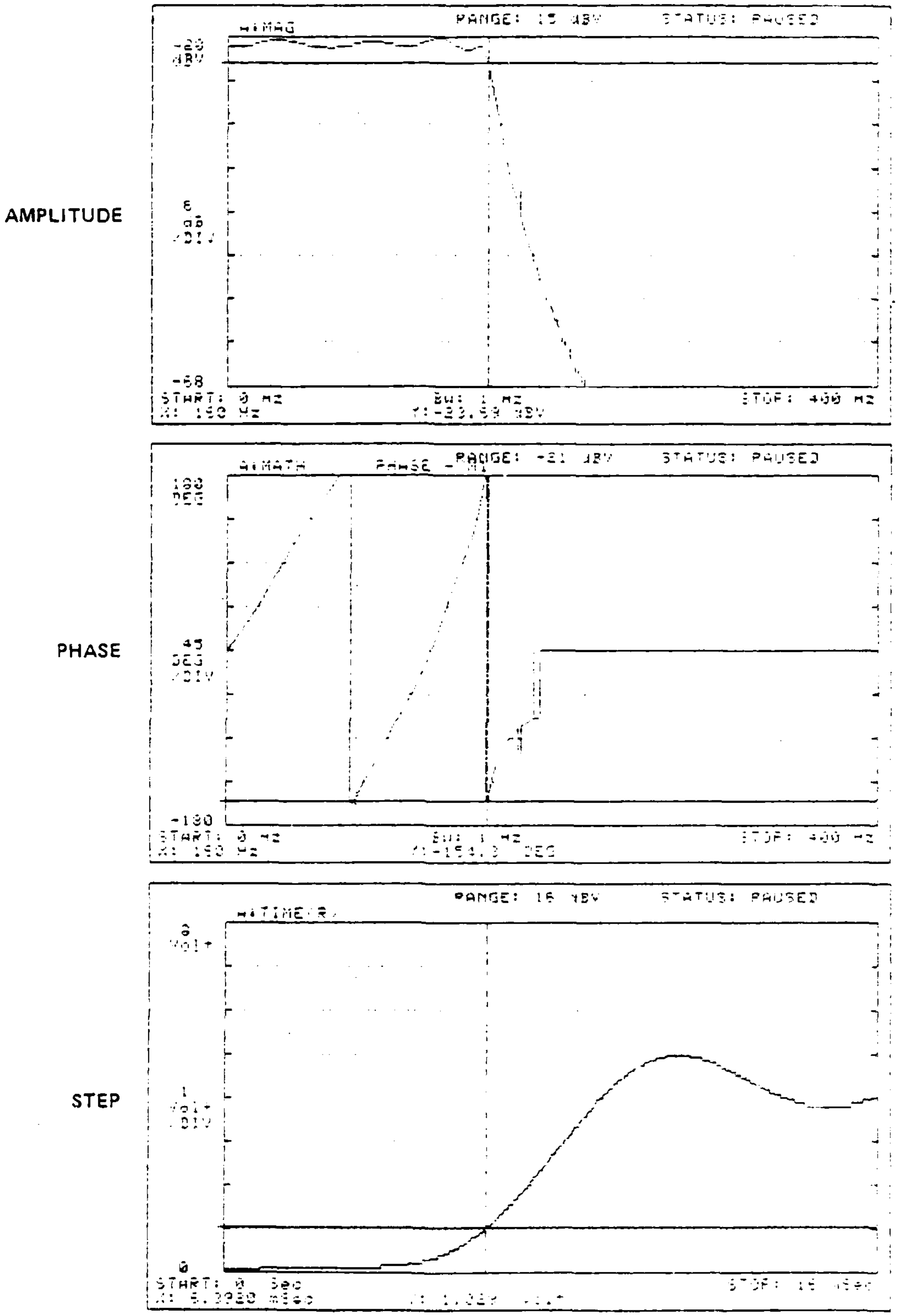


FIGURE A-21. 8-POLE 1dB RIPPLE CHEBYSHEV LOW-PASS RESPONSE

AMPLITUDE

PHASE

STEP

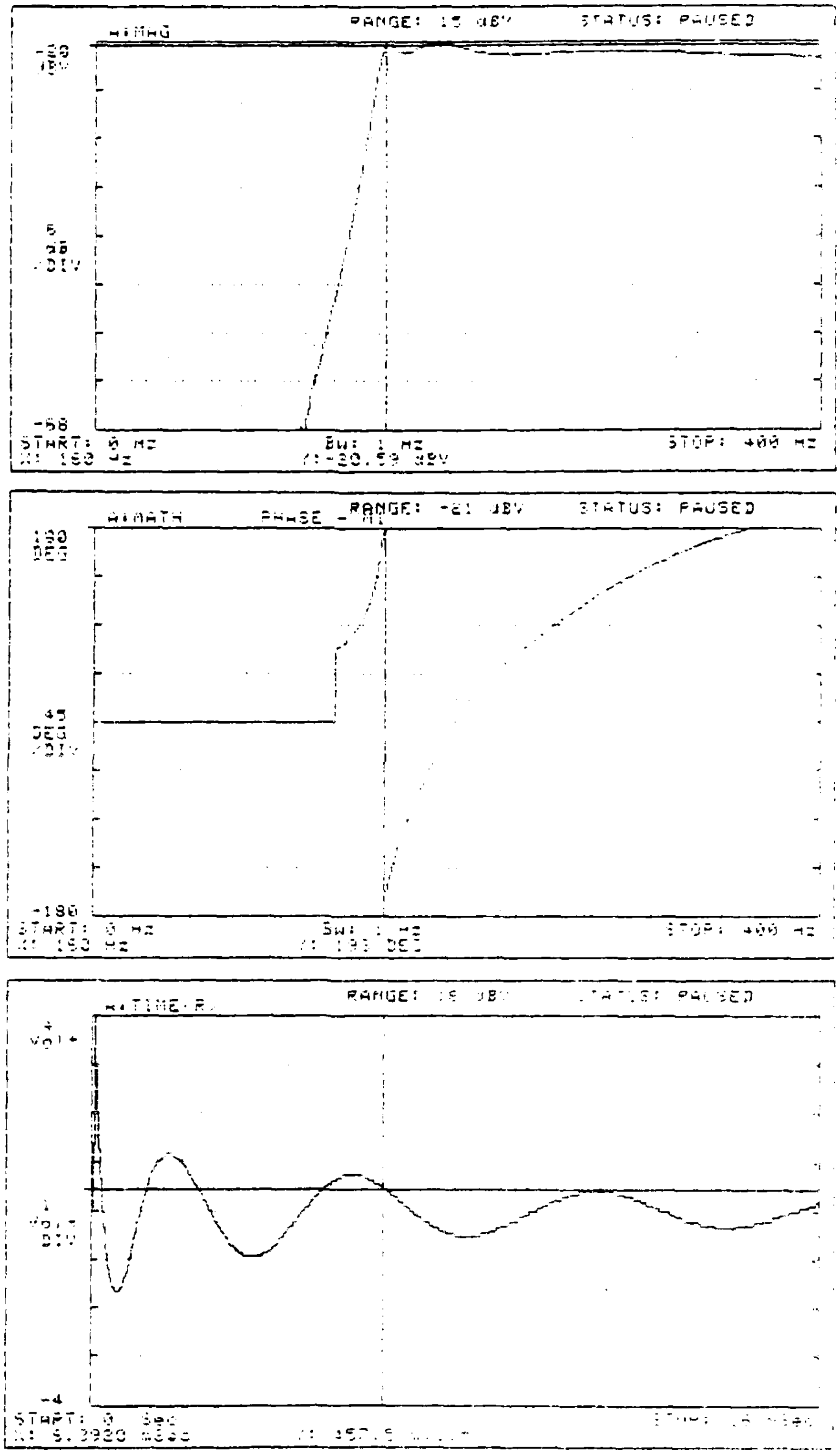


FIGURE A-22. 8-POLE 1dB RIPPLE CHEBYSHEV HIGH-PASS RESPONSE

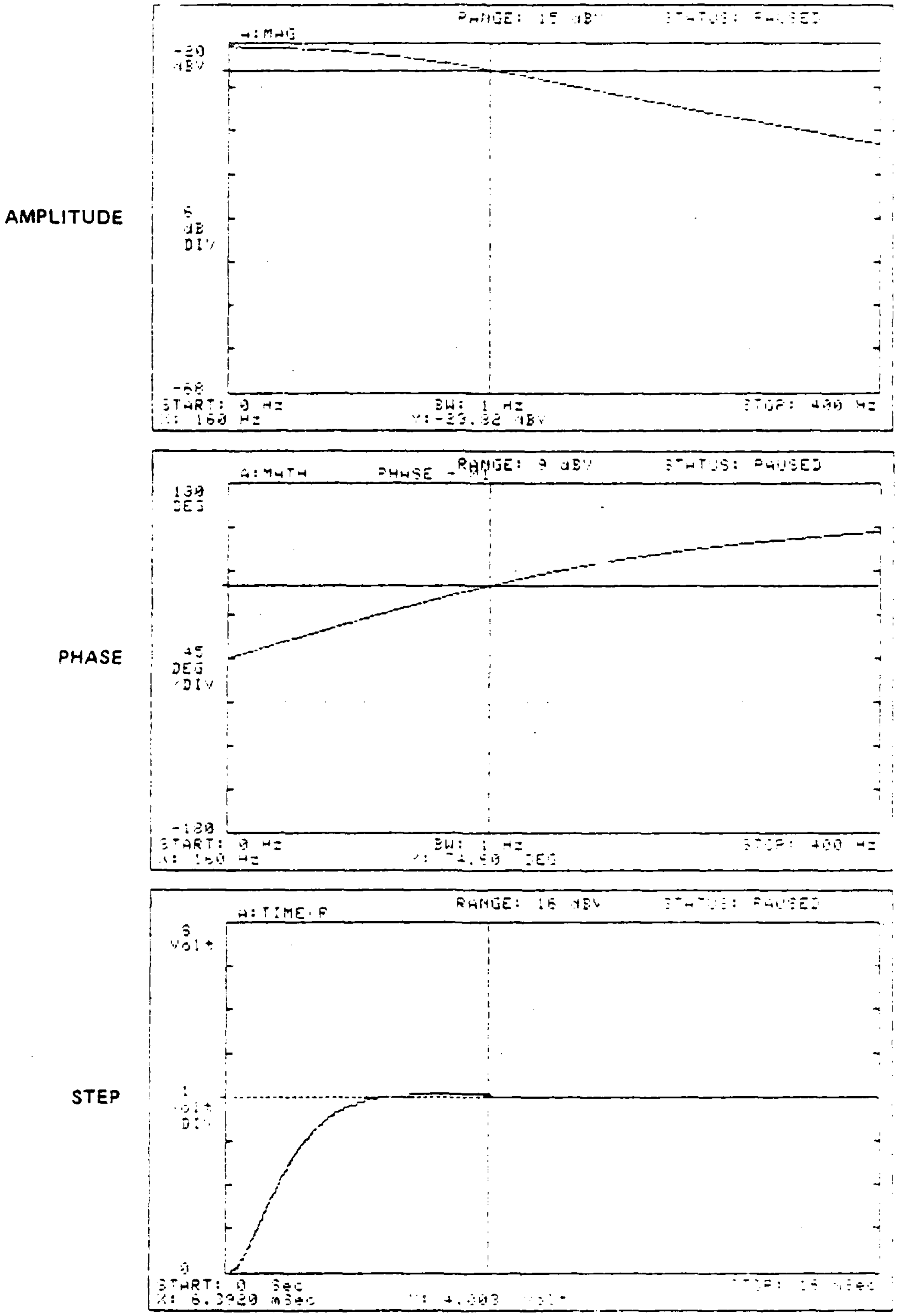


FIGURE A-23. 2-POLE BESSEL LOW-PASS RESPONSE

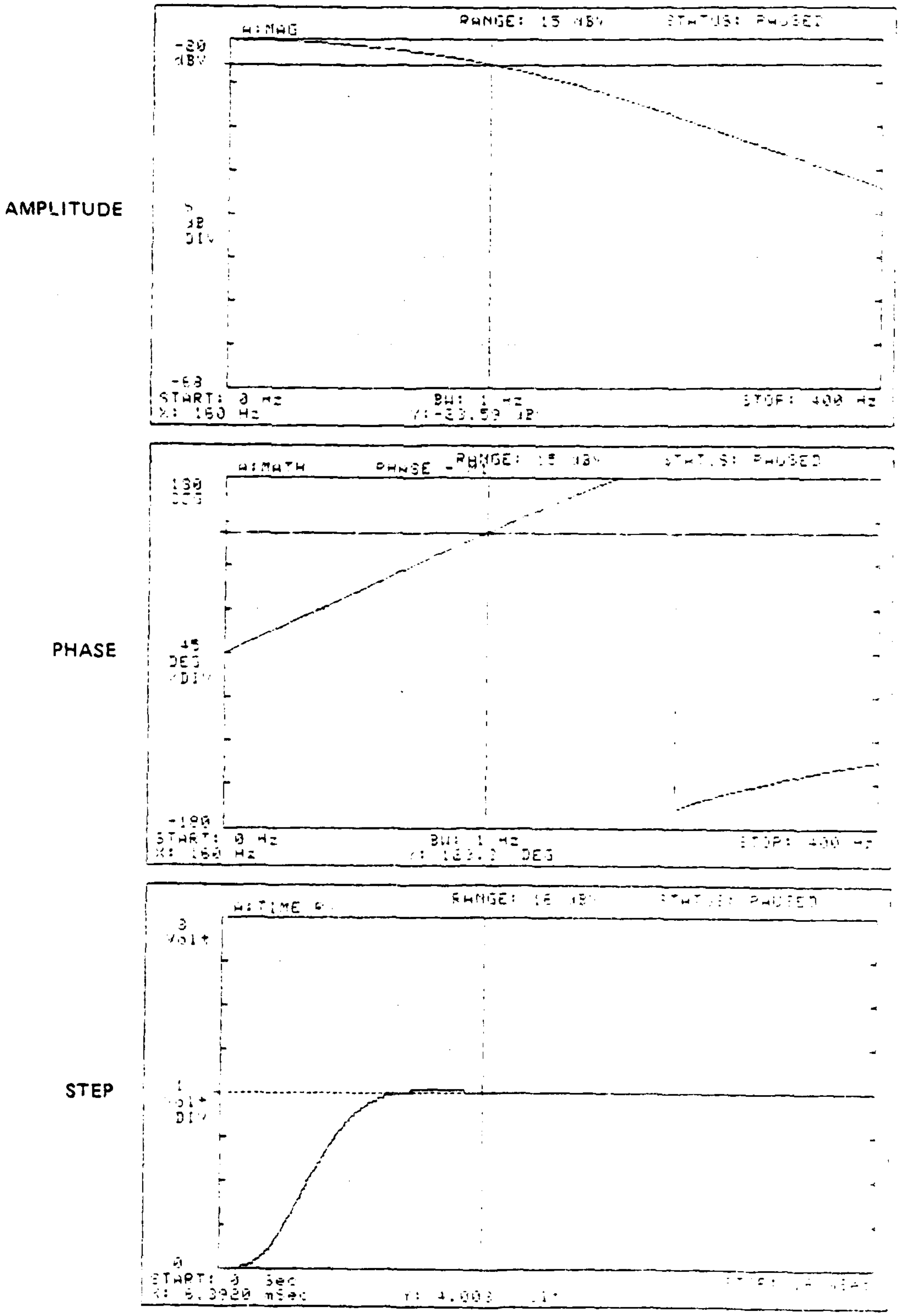


FIGURE A-24. 4-POLE BESSEL LOW-PASS RESPONSE

AMPLITUDE

PHASE

STEP

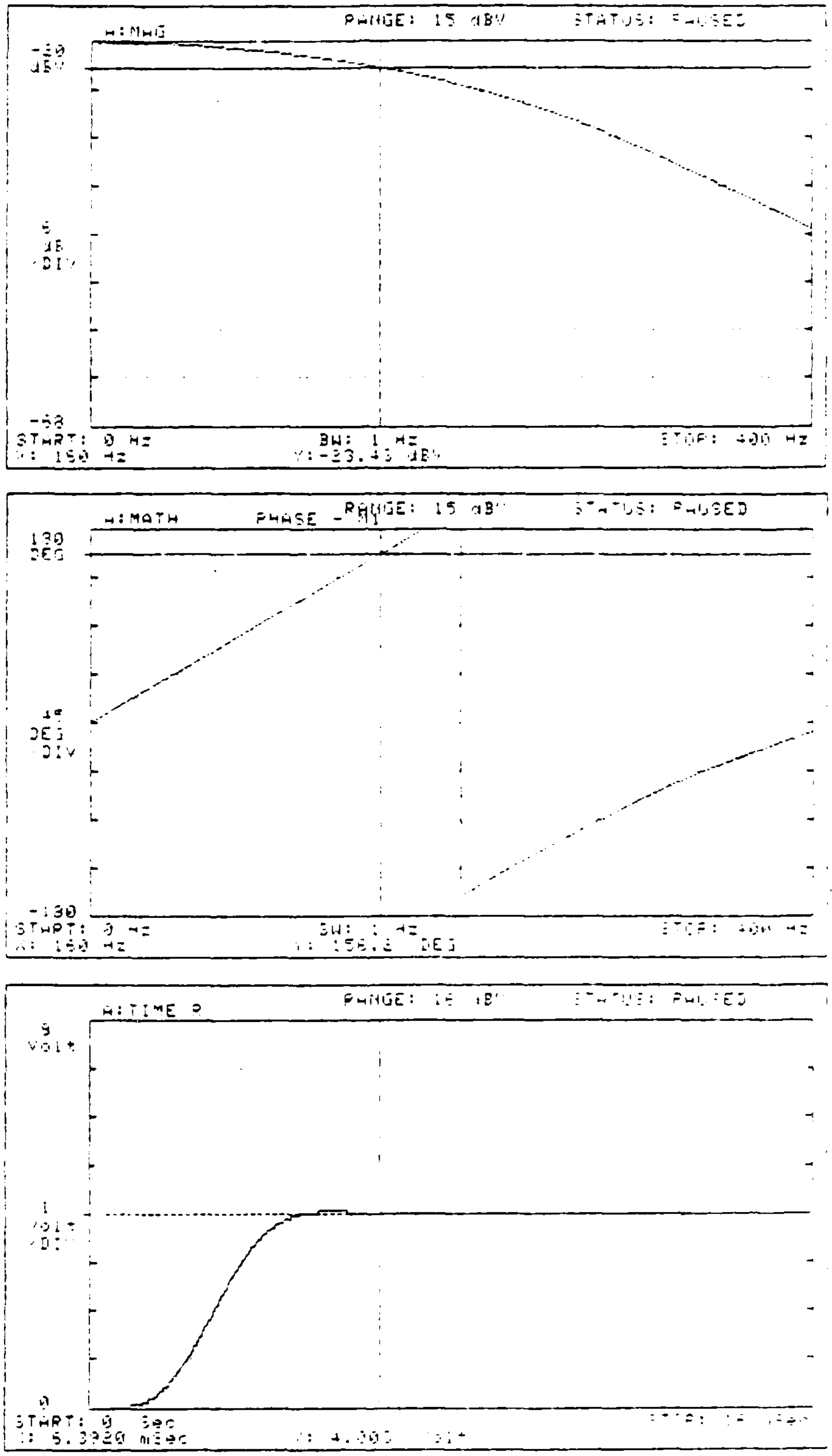


FIGURE A-25. 6-POLE BESSEL LOW-PASS RESPONSE

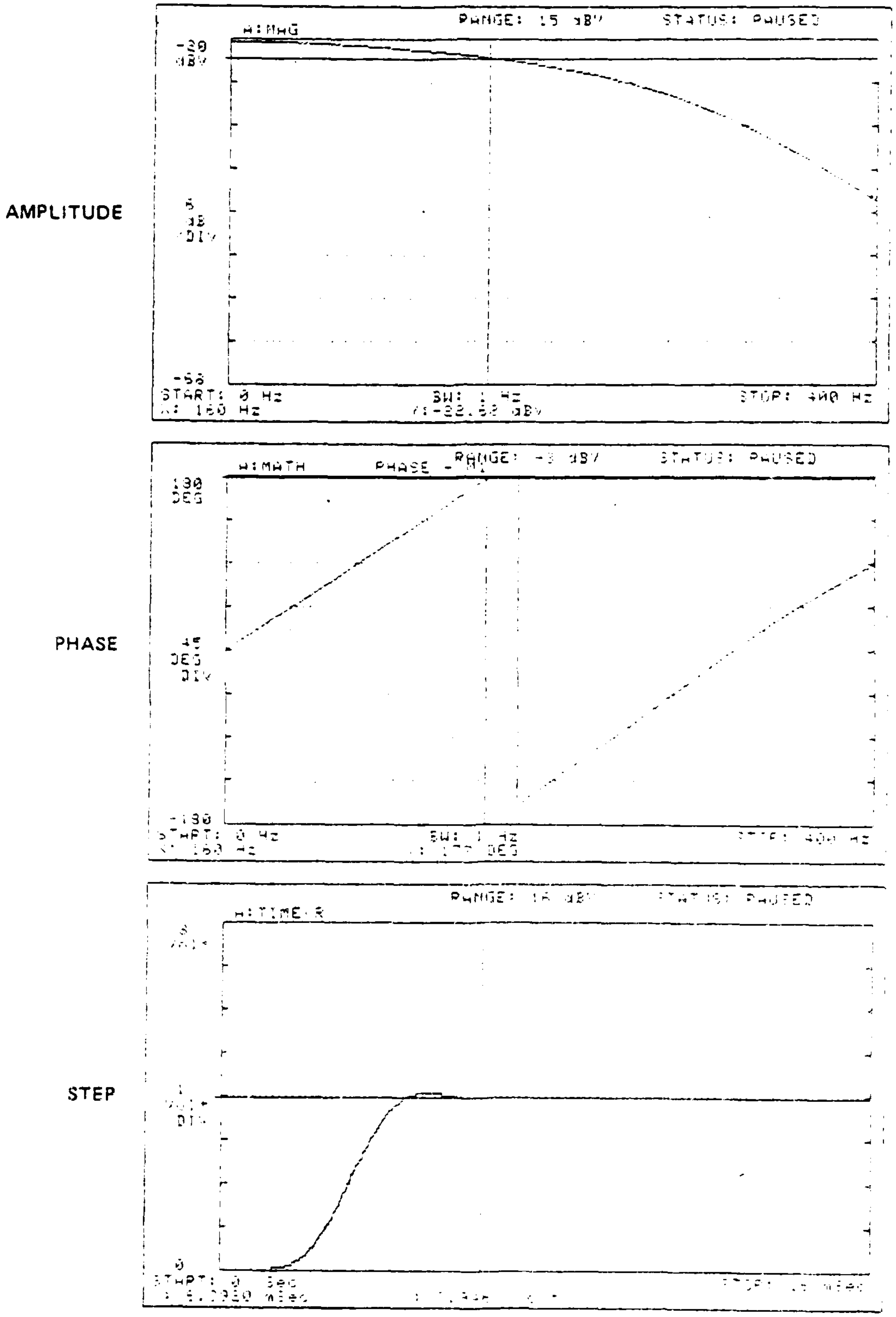


FIGURE A-26. 8-POLE BESSEL LOW-PASS RESPONSE

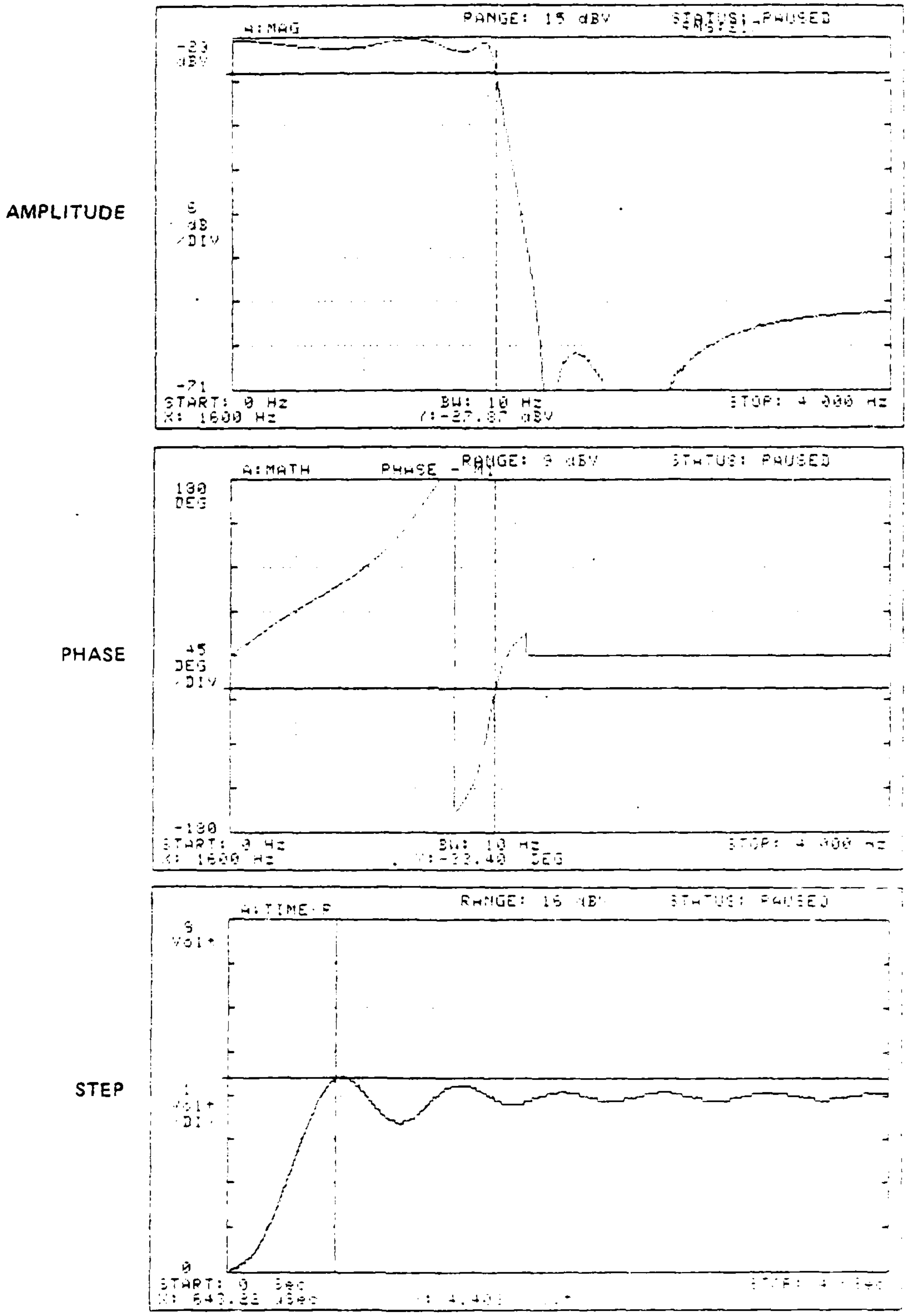


FIGURE A-27. 5-POLE 4-ZERO 1.25dB/39dB ELLIPTIC LOW-PASS RESPONSE

AMPLITUDE

PHASE

STEP

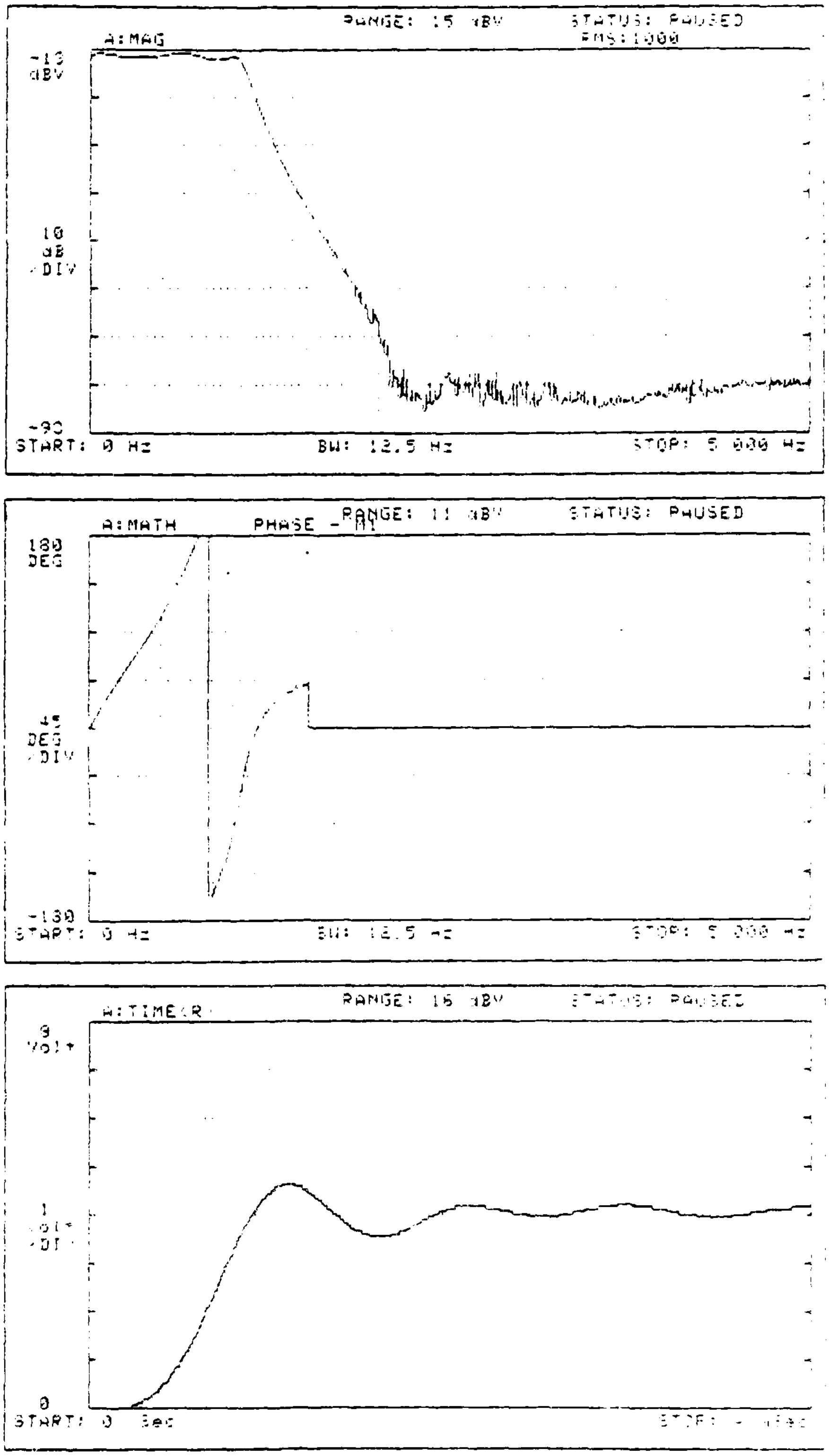


FIGURE A-28. 5-POLE 4-ZERO 1dB/69dB ELLIPTIC LOW-PASS RESPONSE

AMPLITUDE

PHASE

STEP

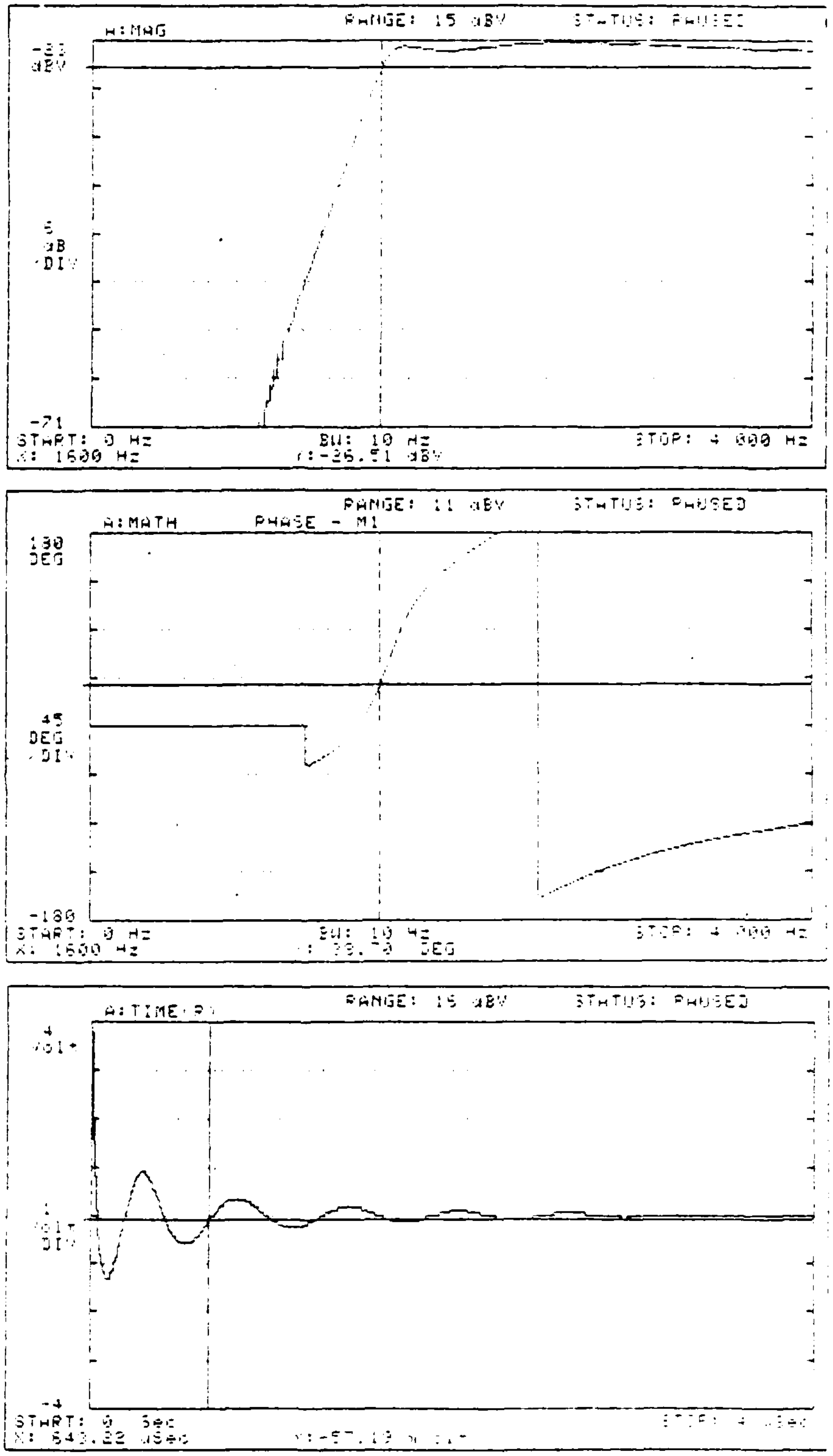


FIGURE A-29. 5-POLE 4-ZERO 1dB/69dB ELLIPTIC HIGH-PASS RESPONSE

AMPLITUDE

PHASE

STEP

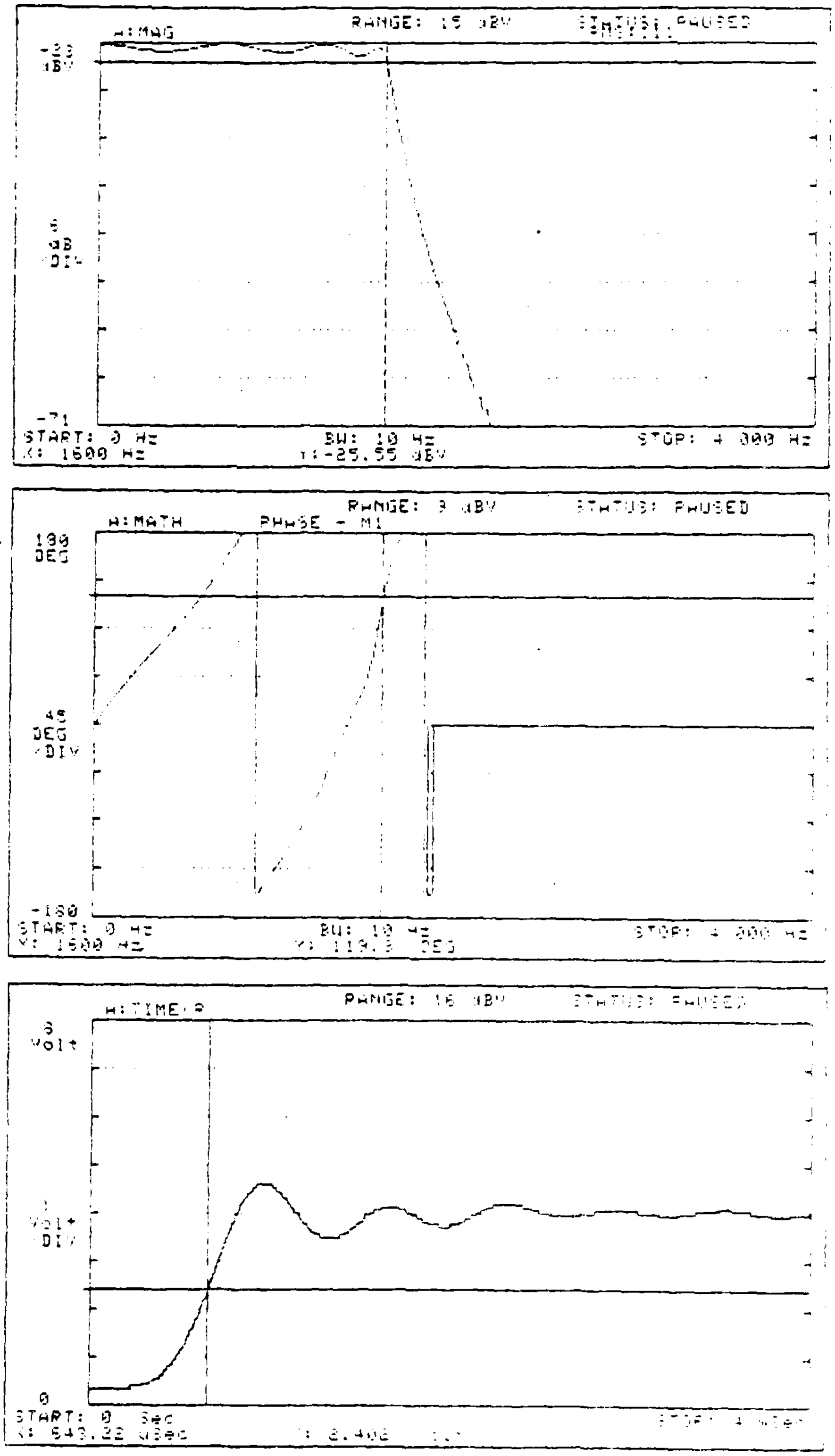


FIGURE A-30. 7-POLE 6-ZERO 1dB/104dB ELLIPTIC LOW-PASS RESPONSE

AMPLITUDE 1

AMPLITUDE 2

AMPLITUDE 3

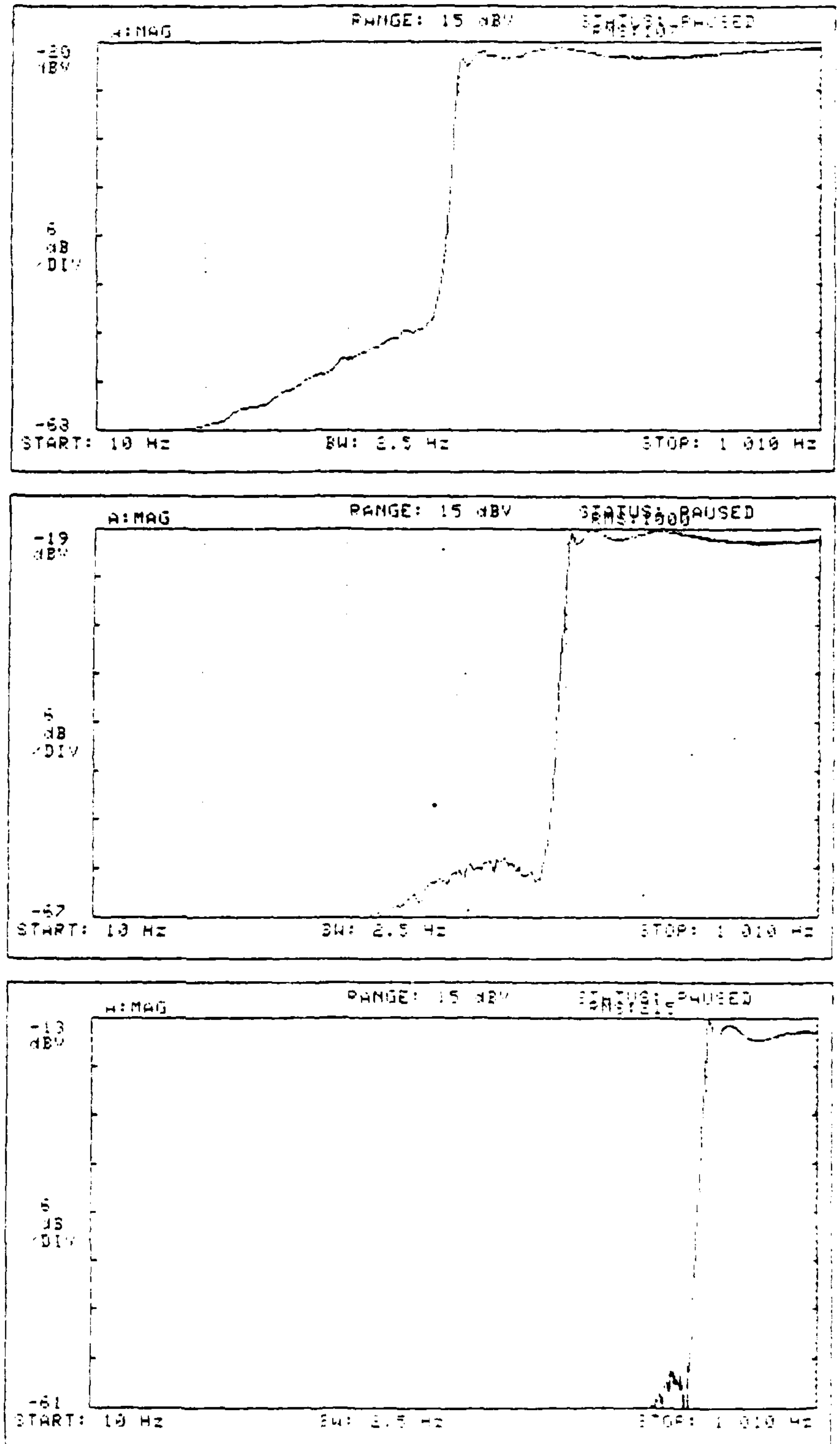


FIGURE A-31. SWITCHED-CAPACITOR ELLIPTIC RESPONSE

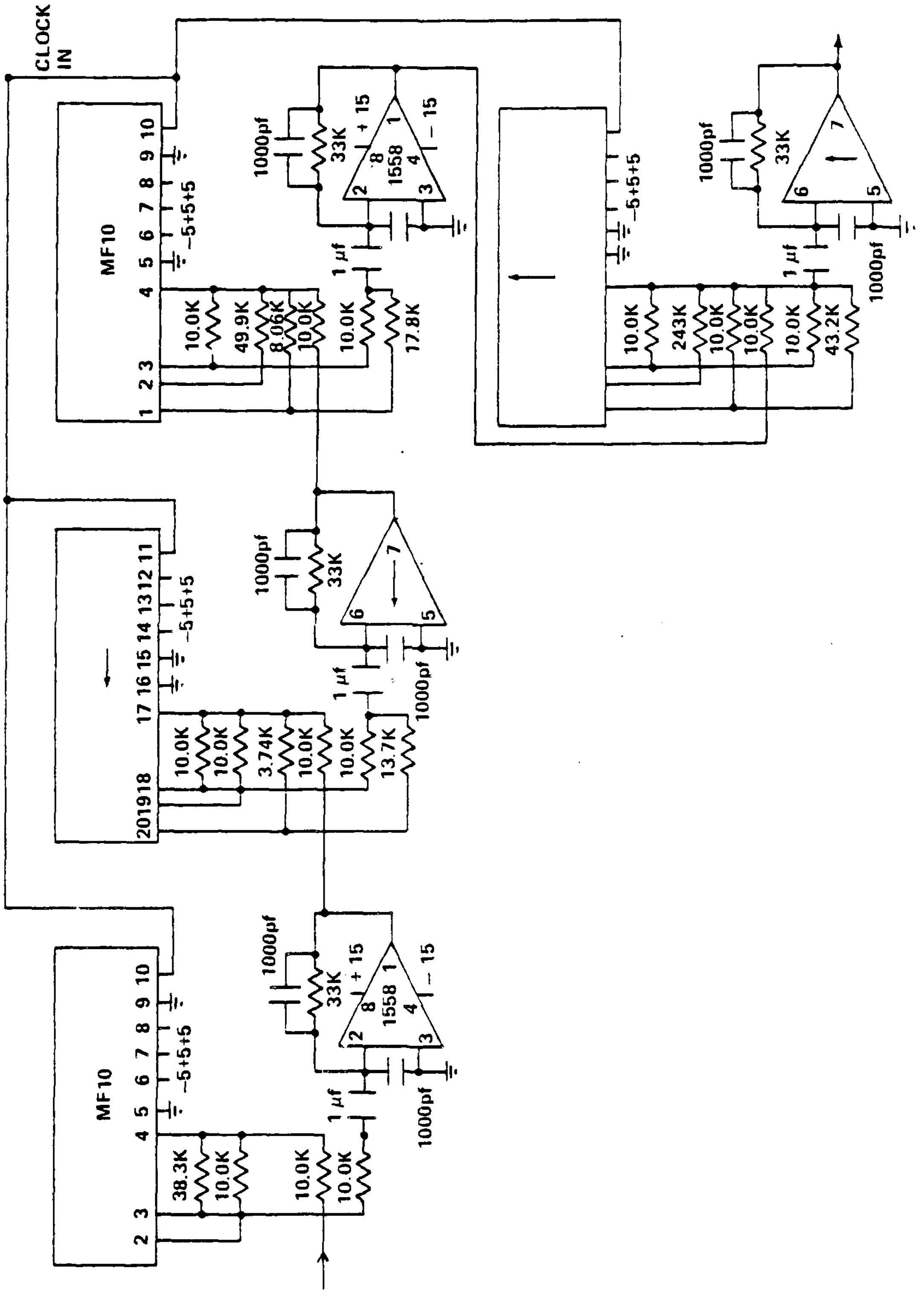


FIGURE A 32. SWITCHED-CAPACITOR ELLIPTIC CIRCUIT

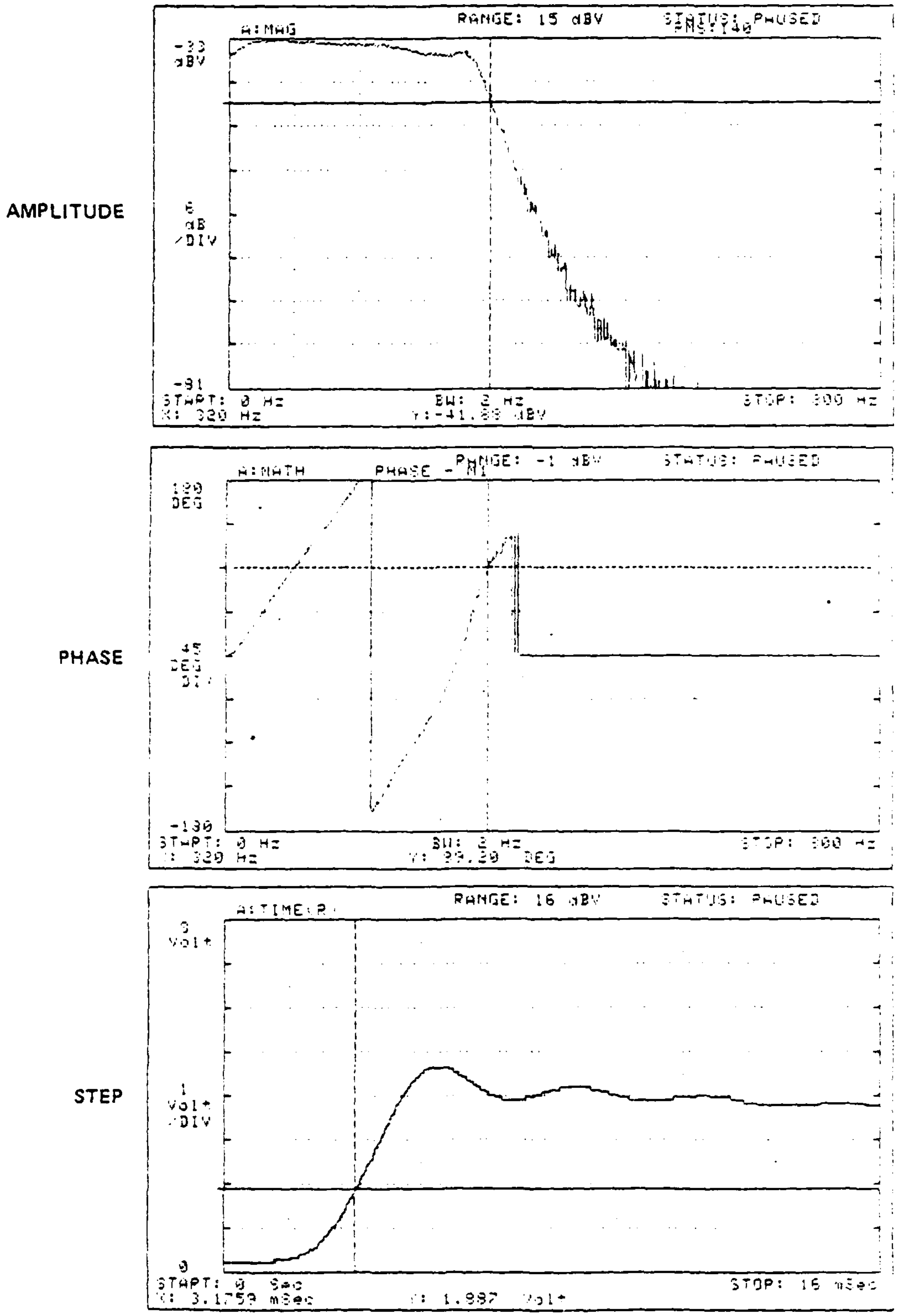


FIGURE A-33. 7-POLE CONSTANT-K LOW-PASS RESPONSE

AMPLITUDE

PHASE

STEP

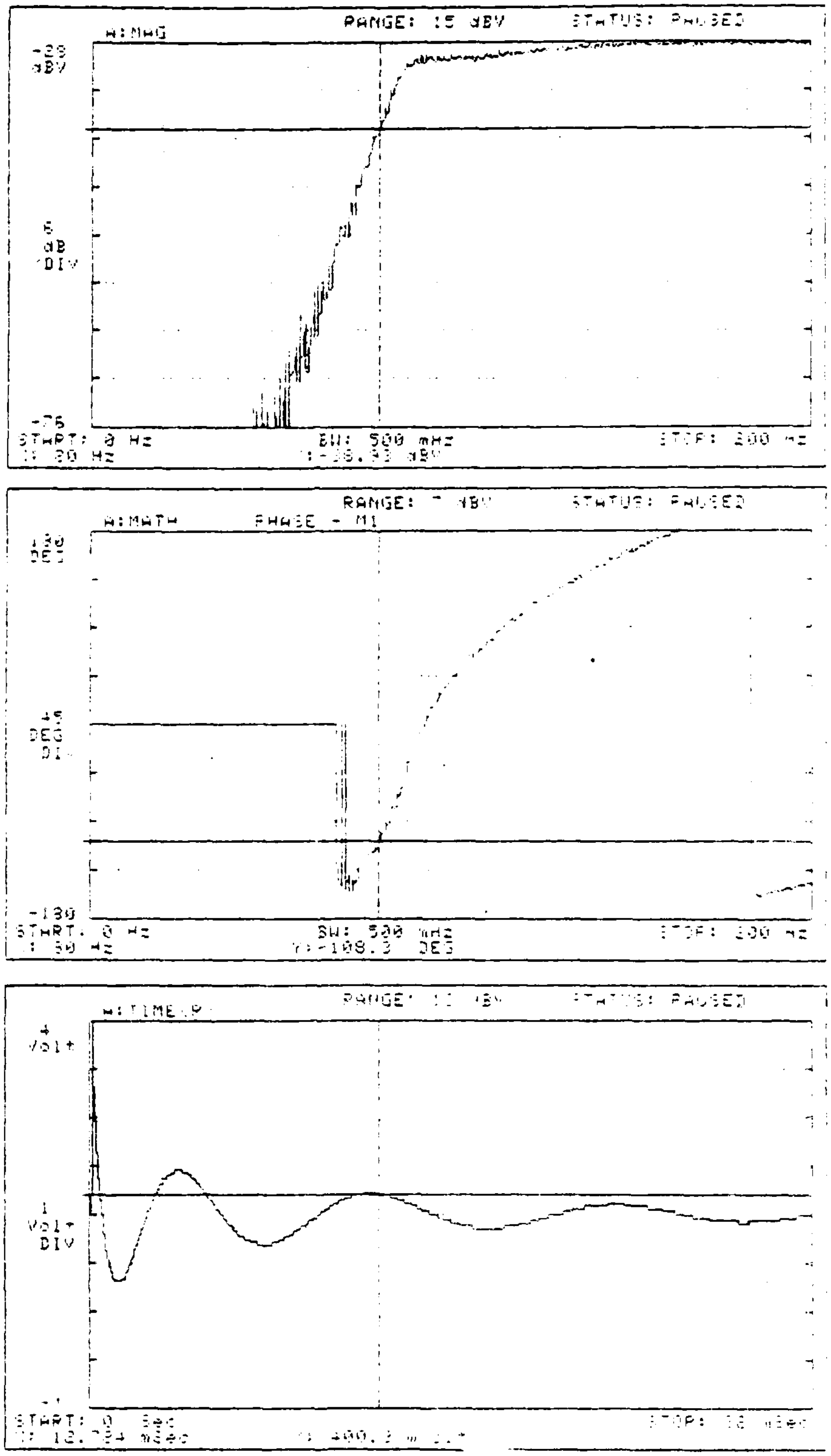


FIGURE A-34. 7-POLE CONSTANT-K HIGH-PASS RESPONSE

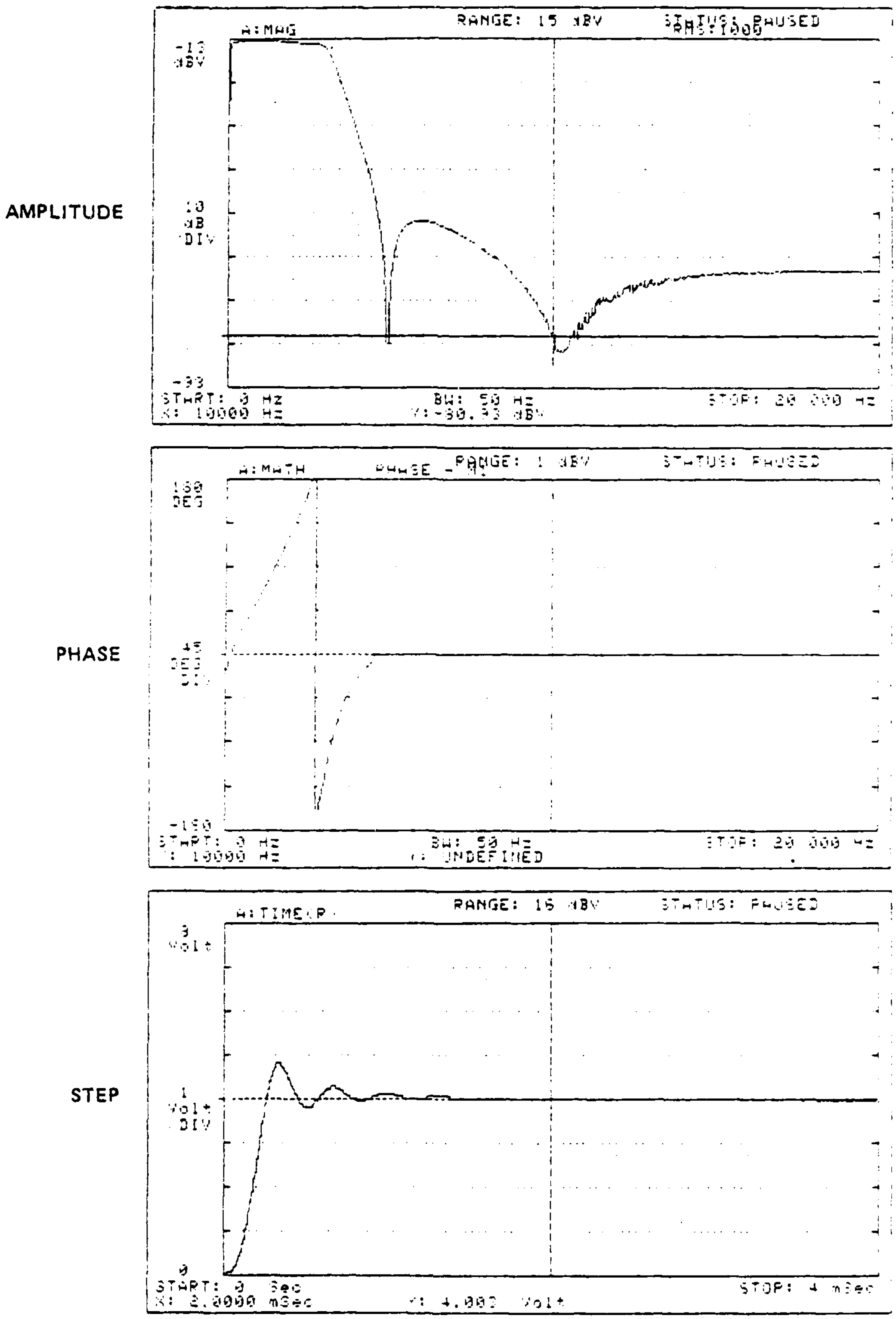
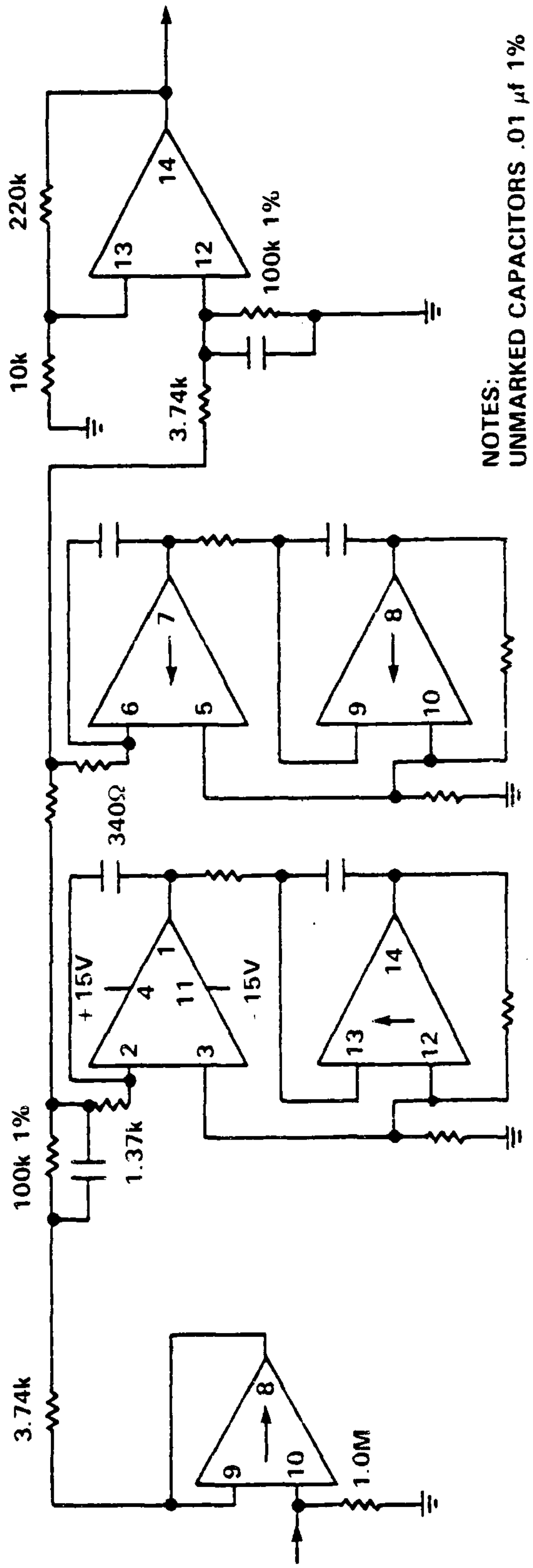


FIGURE A-35. ELLIPTIC-LIKE RESPONSE



NOTES:
UNMARKED CAPACITORS .01 μf 1%
UNMARKED RESISTORS 7.5k 1%
OP AMPS LF 347

FIGURE A-36 ELLIPTIC-LIKE CIRCUIT

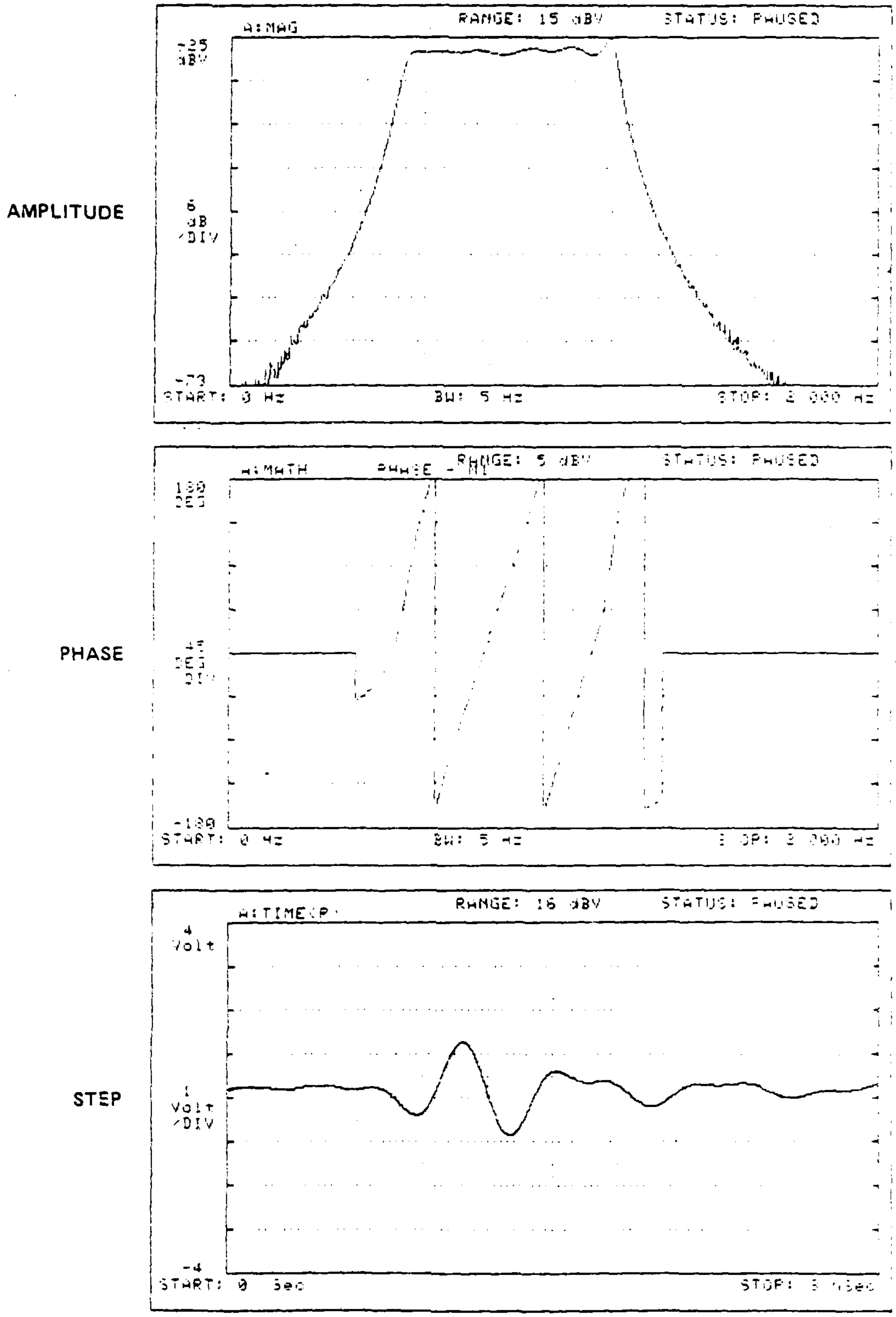


FIGURE A-37. 12-POLE LERNER BANDPASS RESPONSE

AMPLITUDE

PHASE

STEP

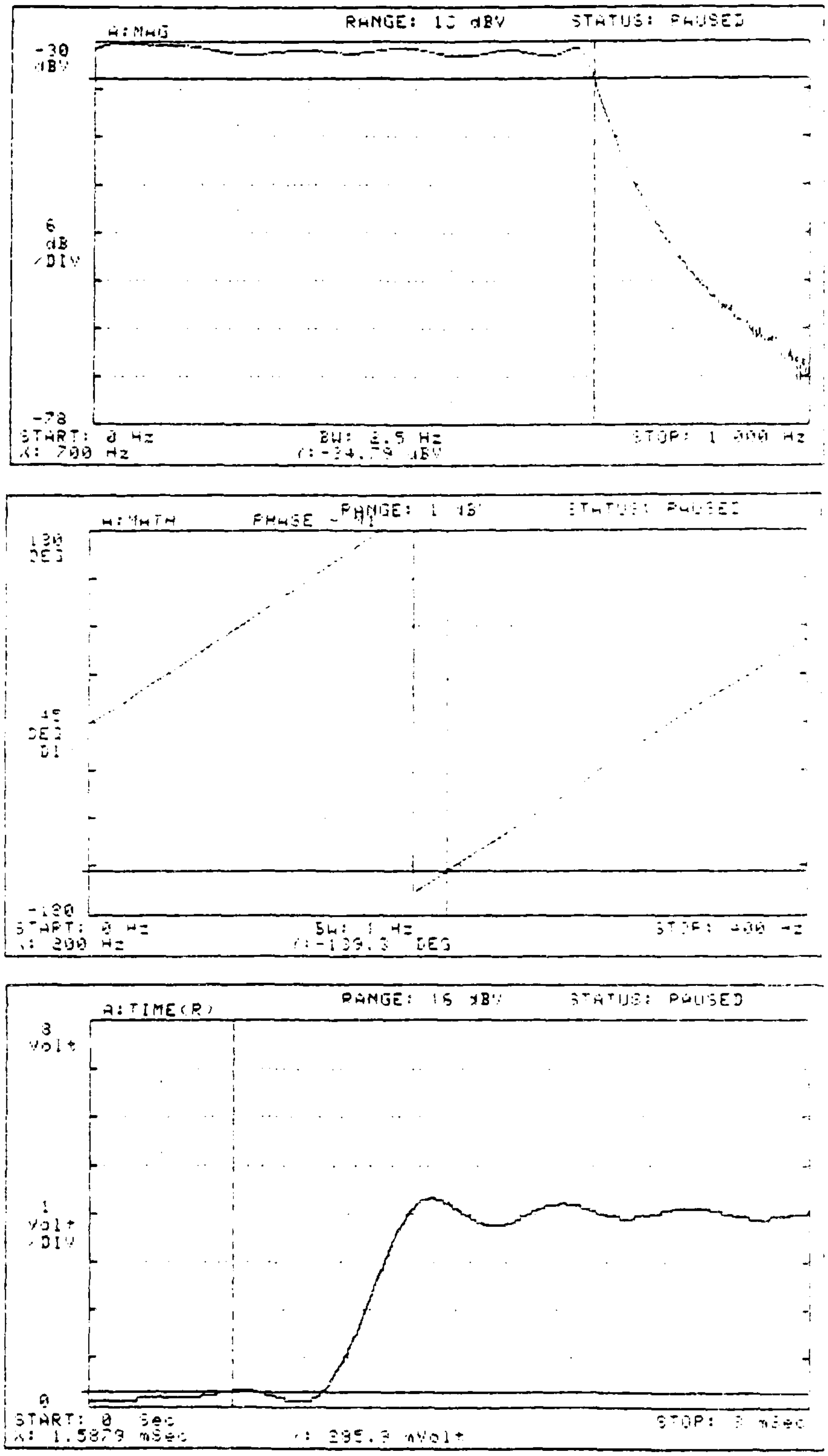


FIGURE A-38. 11-POLE LERNER LOW-PASS RESPONSE

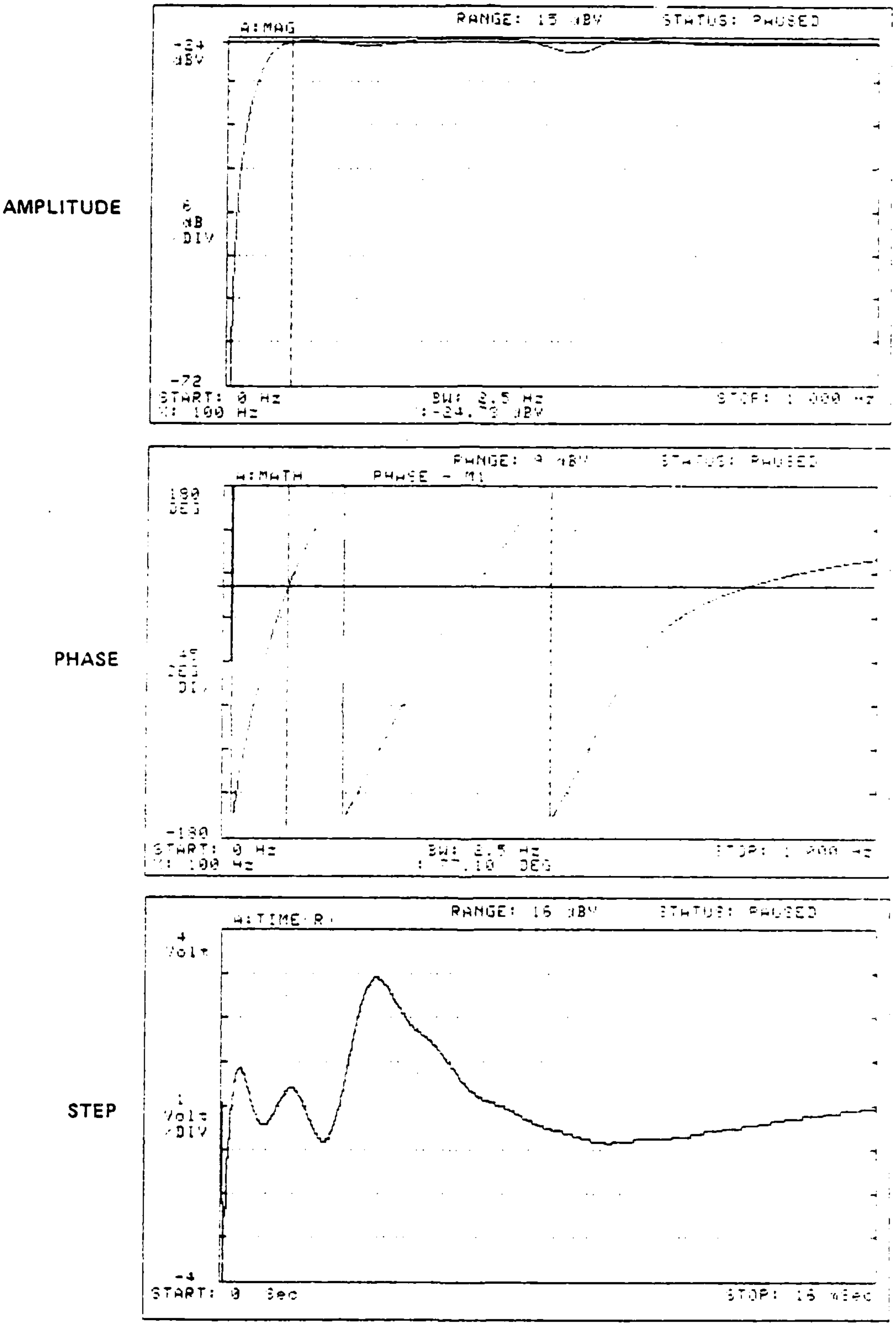


FIGURE A-39. 11-POLE LERNER HIGH-PASS RESPONSE

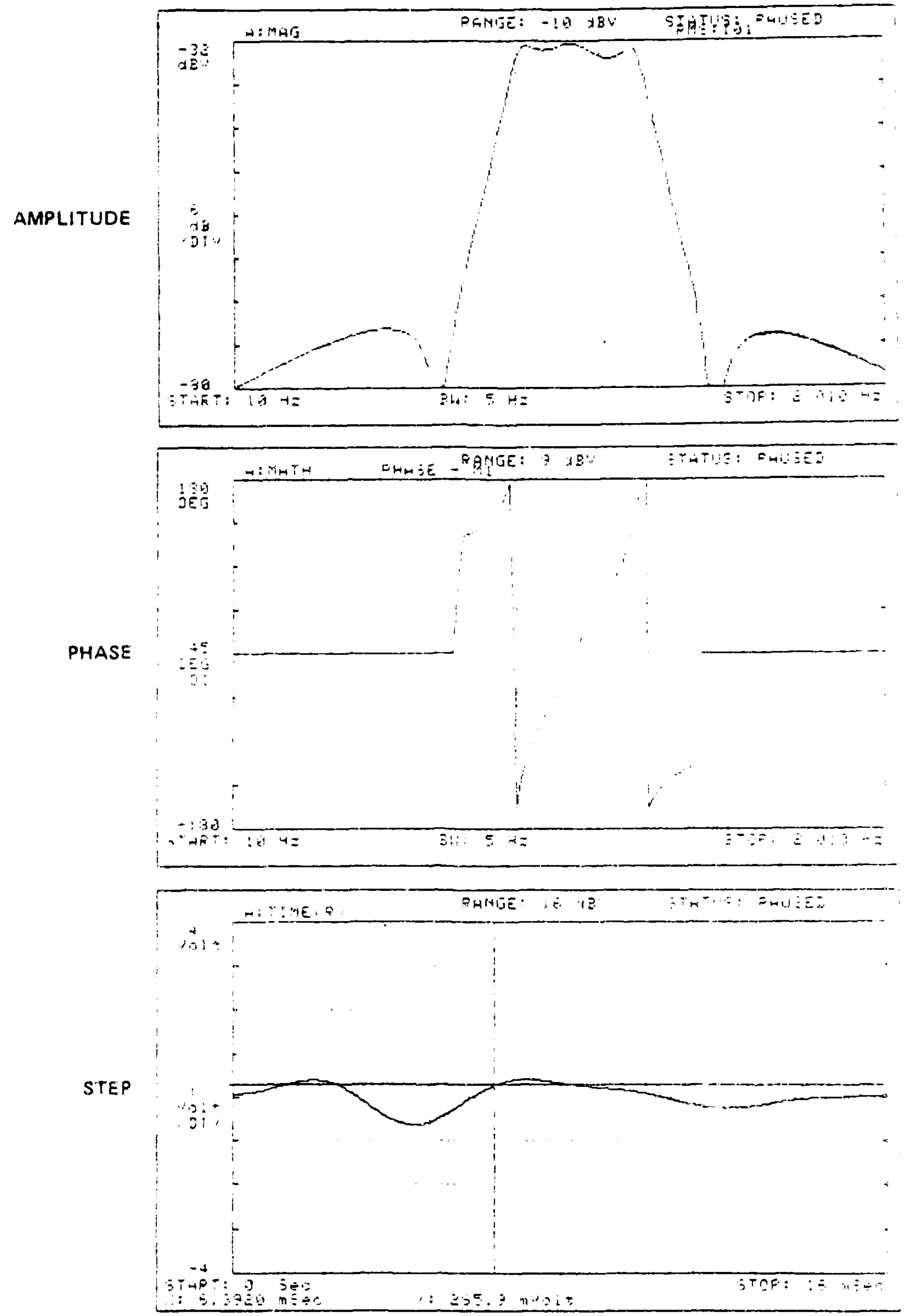


FIGURE A-40. 6-POLE 4-ZERO 1.25dB/39dB ELLIPTIC BANDPASS RESPONSE

AMPLITUDE

PHASE

STEP

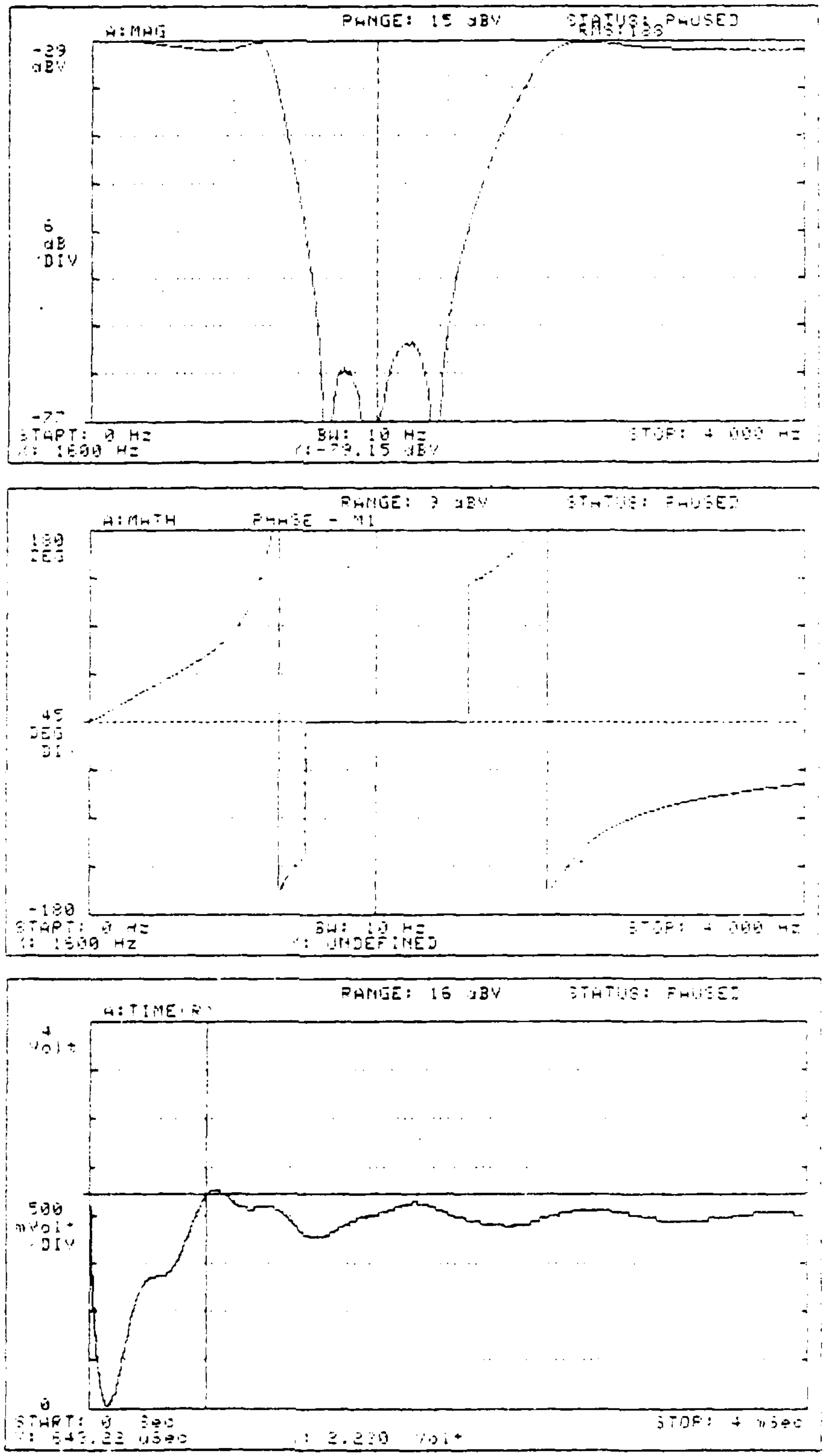


FIGURE A-41. 6-POLE 6-ZERO 1.25dB/39dB ELLIPTIC BAND REJECT RESPONSE

AMPLITUDE

PHASE

STEP

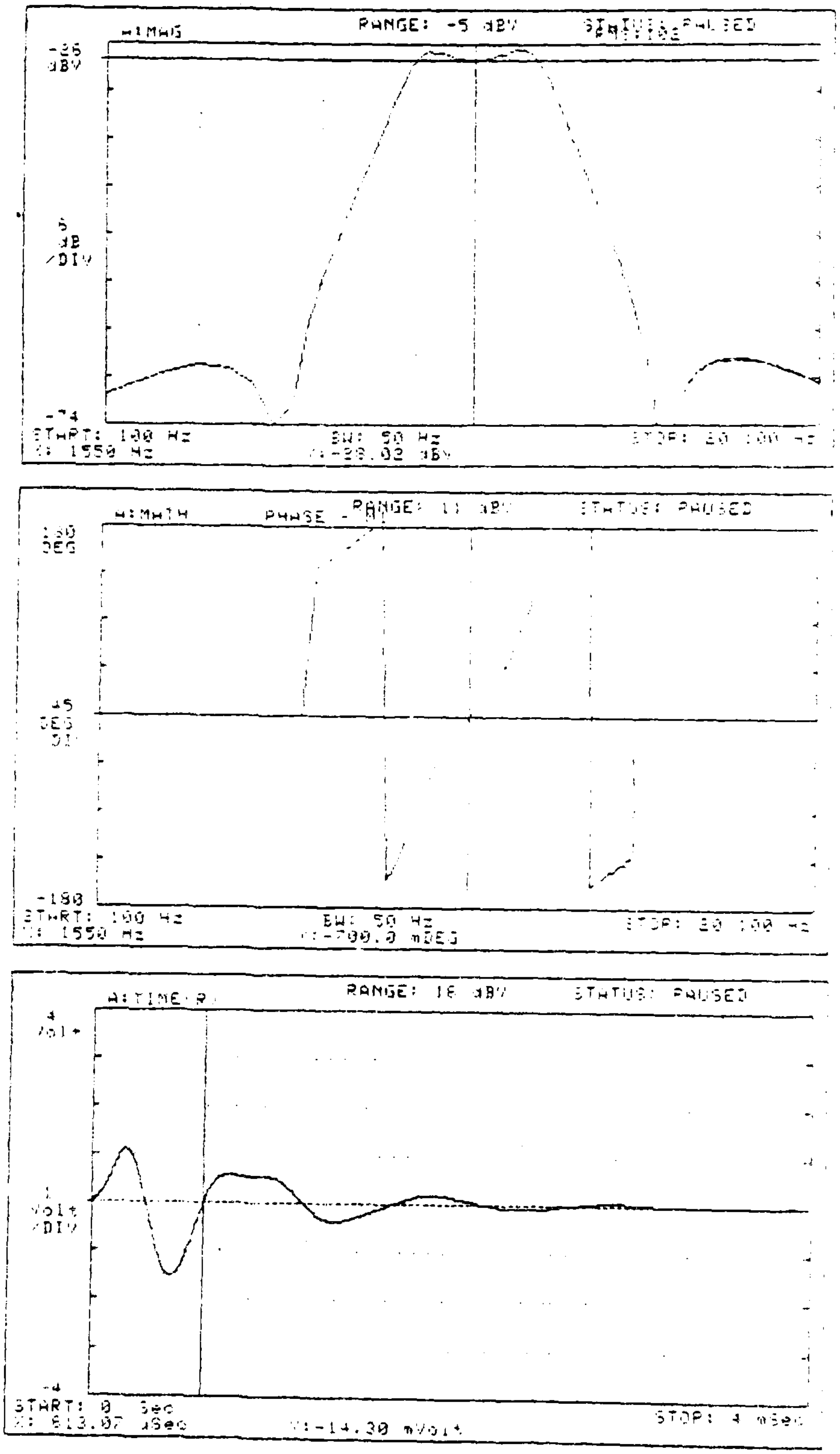


FIGURE A-42. 6-POLE 4-ZERO QUASI-ELLIPTIC BANDPASS RESPONSE

AMPLITUDE

PHASE

STEP

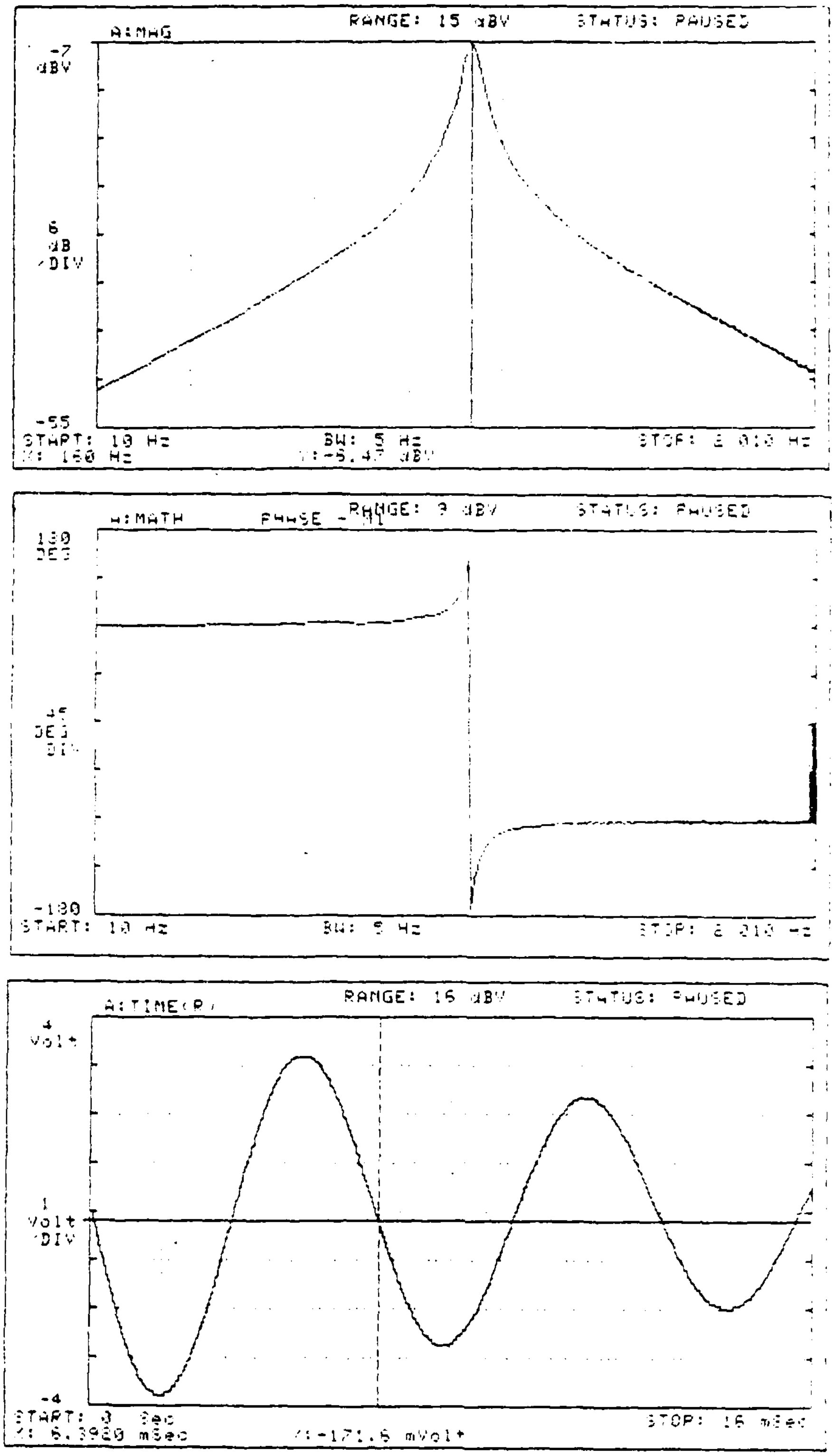


FIGURE A-43. Q-OF-10 NARROWBAND RESPONSE

AMPLITUDE

PHASE

STEP

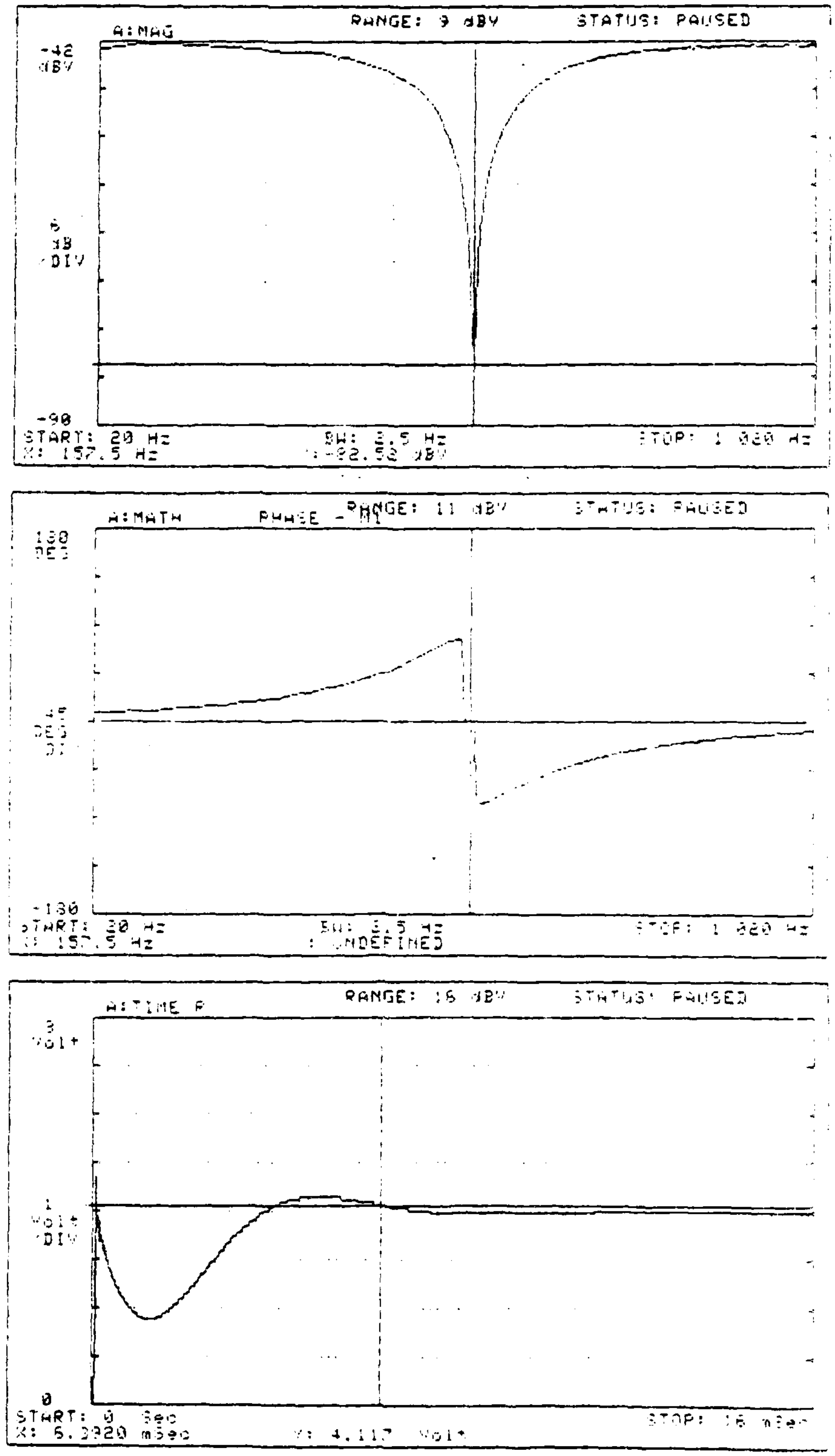


FIGURE A-44. Q-OF-3 NOTCH RESPONSE

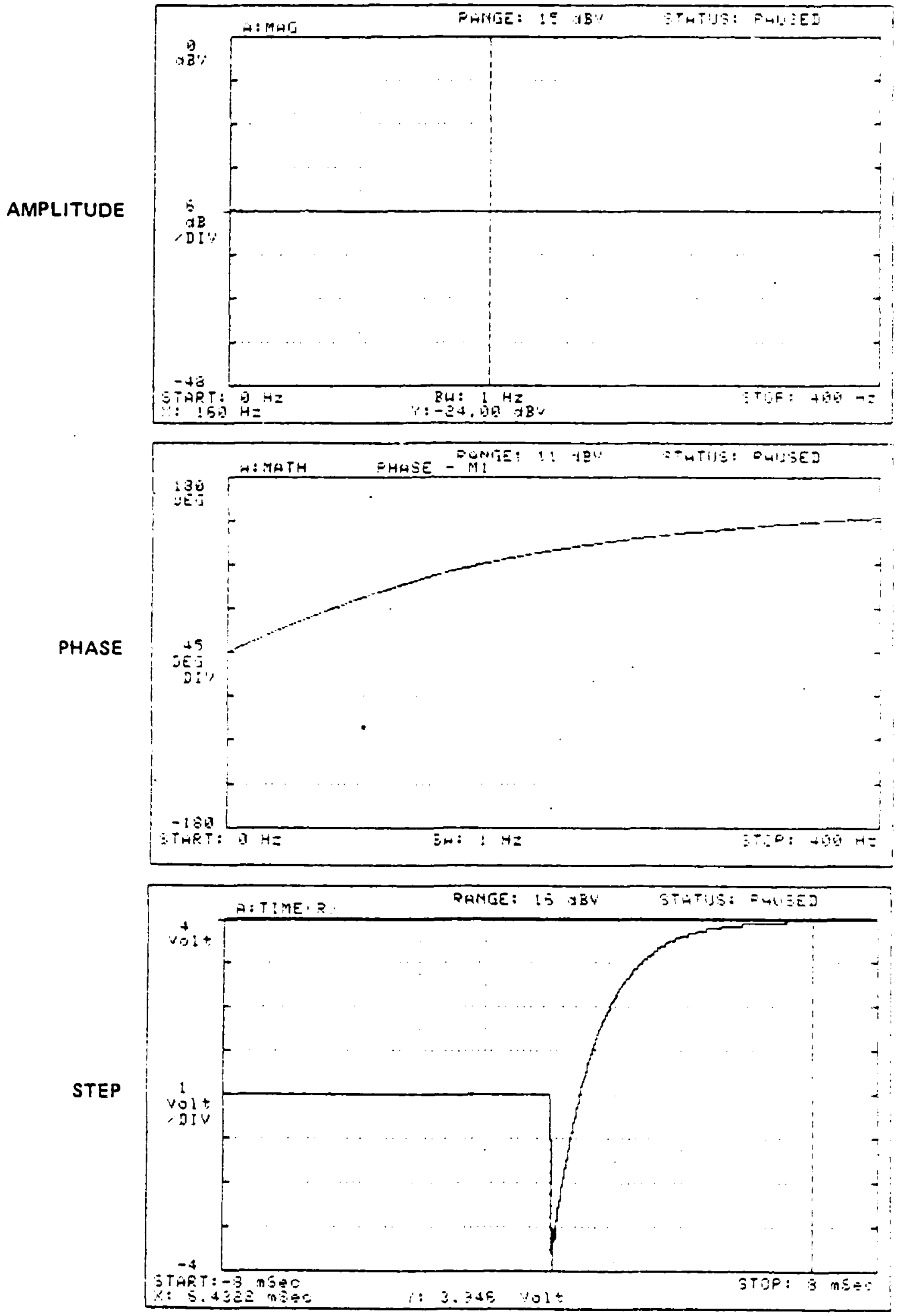


FIGURE A-45. 1-POLE 1-ZERO ALL-PASS RESPONSE

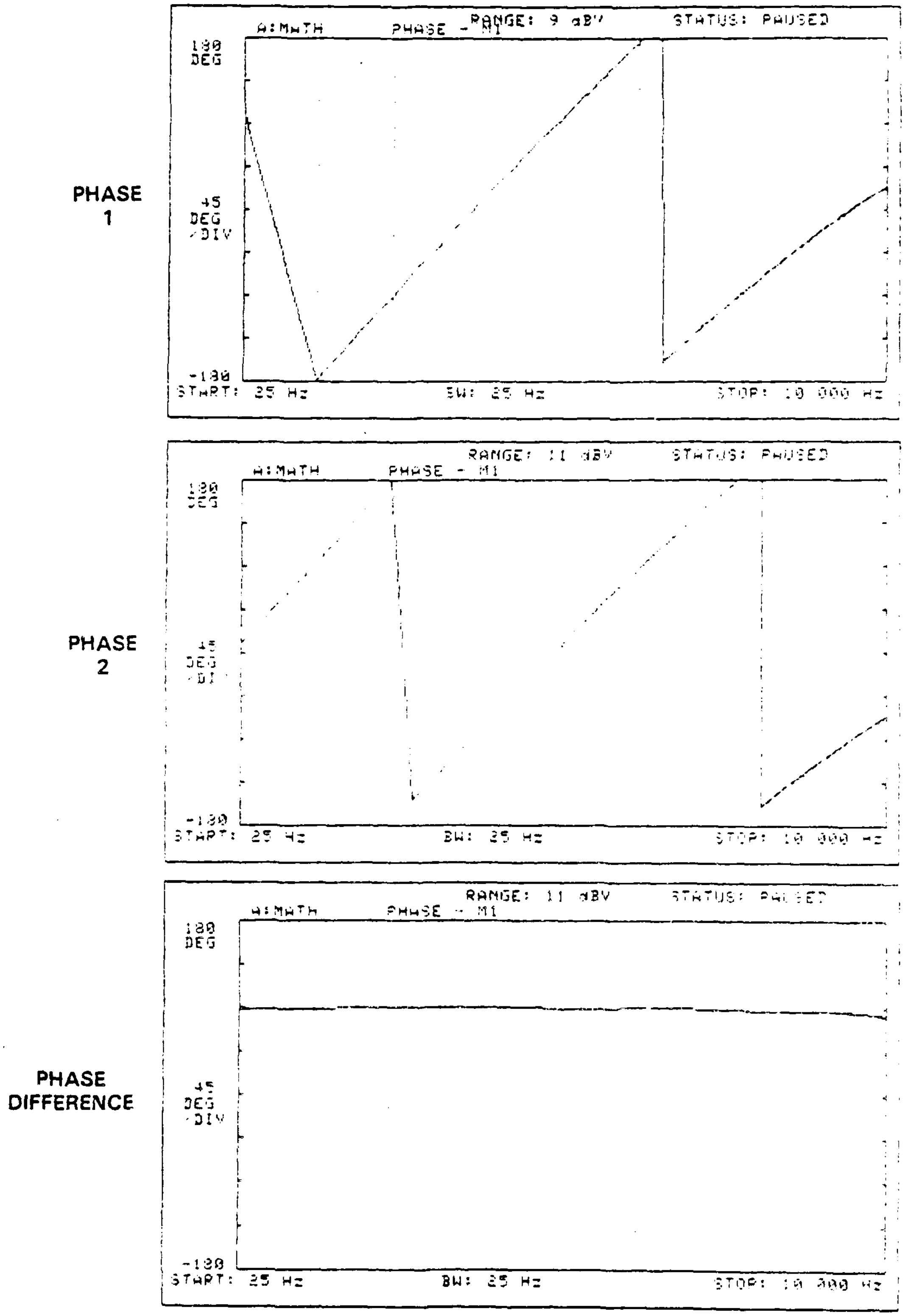


FIGURE A-46. 2-POLE 2-ZERO 90 DEGREE-PHASE-DIFFERENCE NETWORK RESPONSE

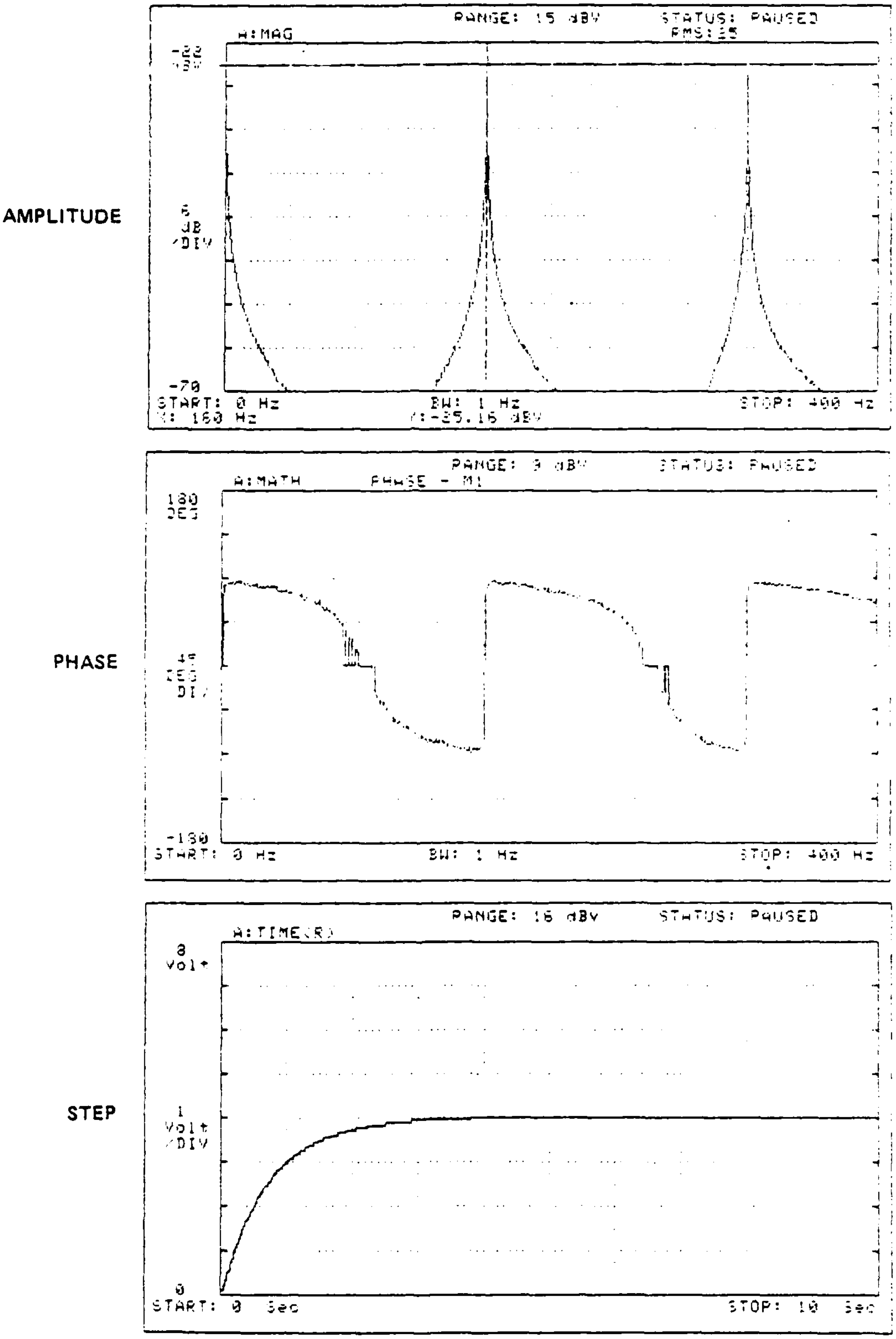


FIGURE A-47. Q-OF-400 COMMUTATING BANDPASS RESPONSE

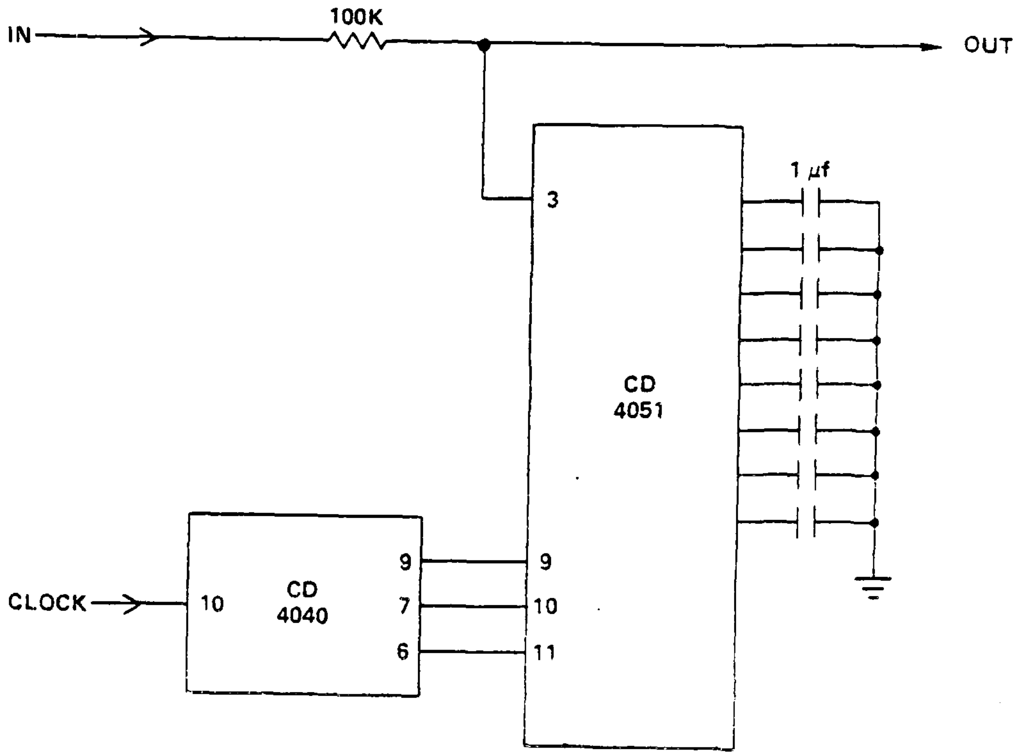


FIGURE A-48. Q-OF-400 COMMUTATING BANDPASS CIRCUIT

AMPLITUDE

PHASE

STEP

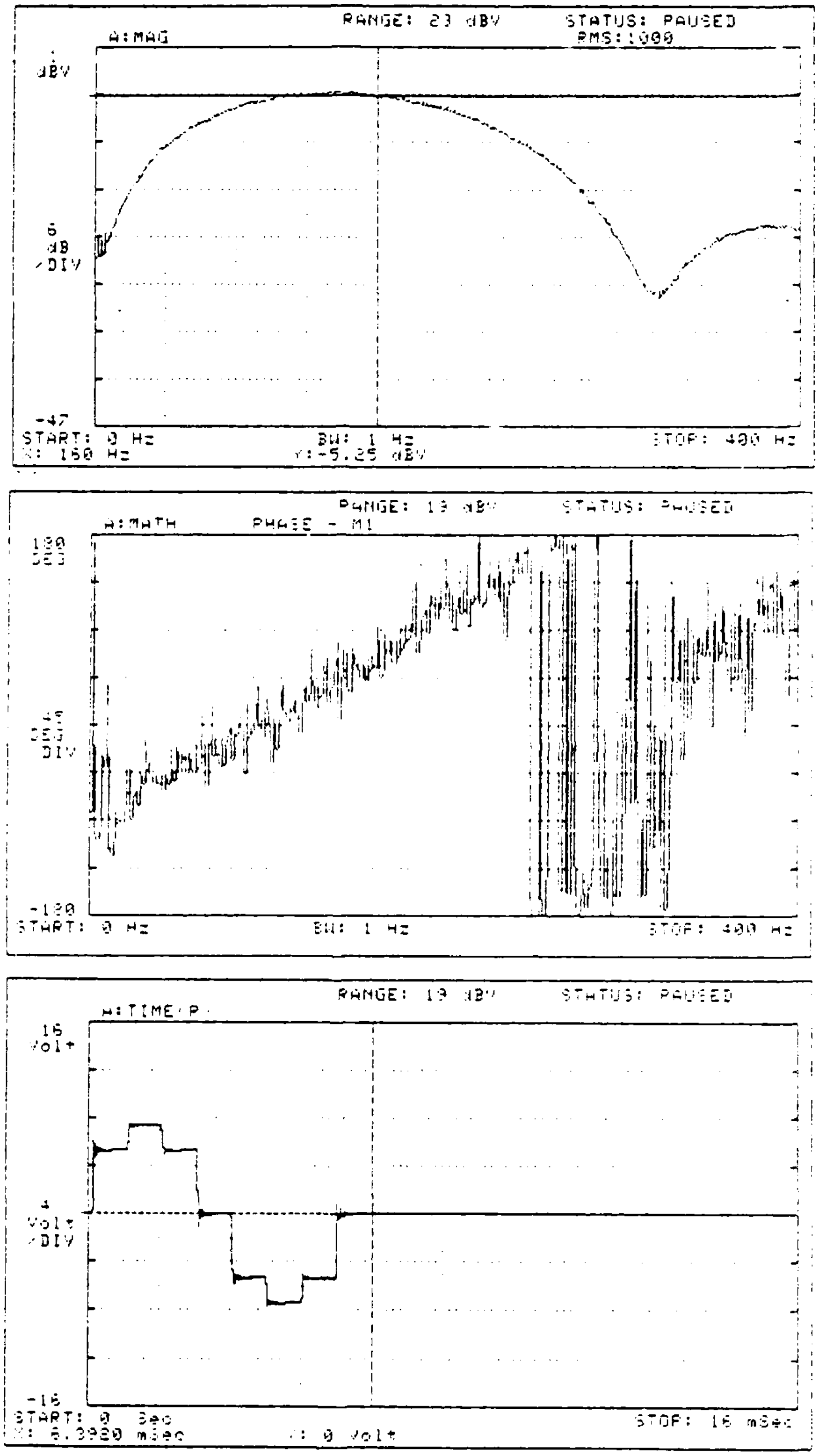


FIGURE A-49. 8-STAGE TRANSVERSAL FILTER RESPONSE

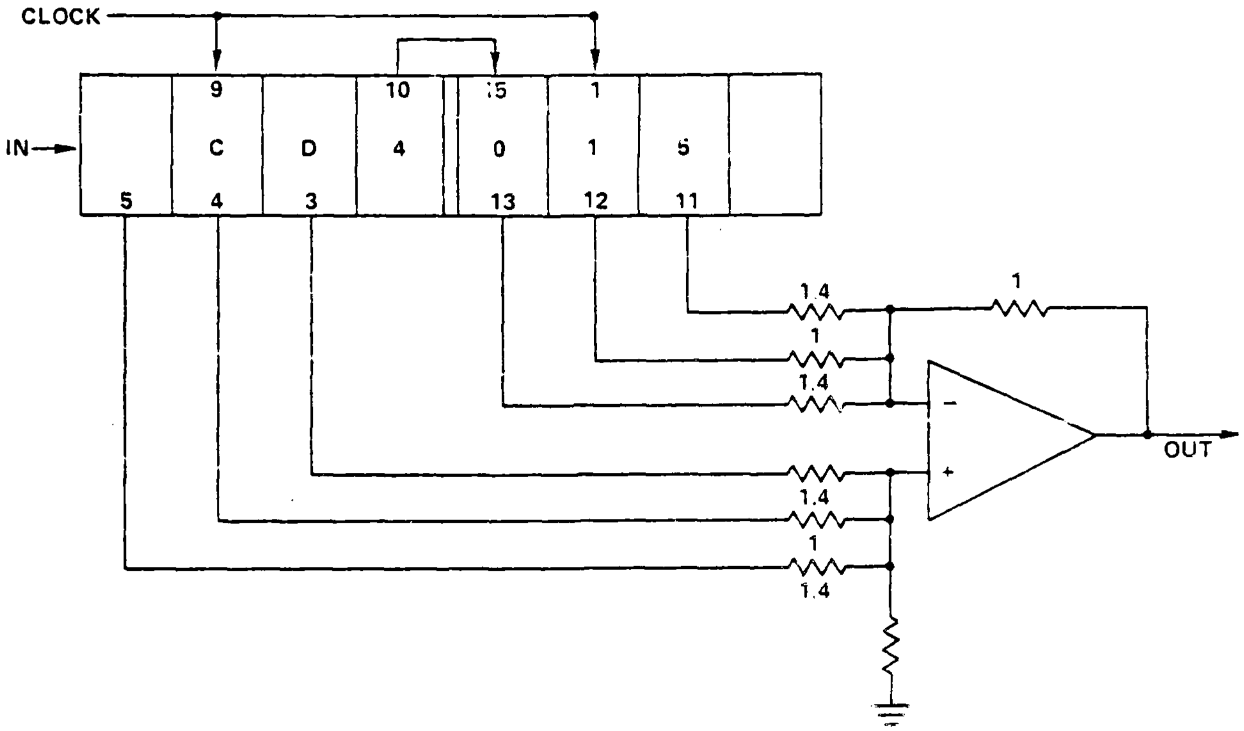


FIGURE A-50. 8-STAGE TRANSVERSAL FILTER CIRCUIT

REFERENCES

A-1. Delagrange, A. D., A Useful Filter Family, NSWC/WOL TR 85-170, 20 Oct 1975.

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